Multiplication of Mixed Numbers

**Objective** To introduce multiplication with mixed numbers.

### Key Concepts and Skills
- Convert between fractions and mixed numbers.  
  [Number and Numeration Goal 5]
- Multiply mixed numbers.  
  [Operations and Computation Goal 5]
- Use the partial-products algorithm to multiply whole numbers, fractions, and mixed numbers.  
  [Operations and Computation Goal 5]
- Recognize the patterns in products when a number is multiplied by a fraction that is less than 1, equal to 1, or greater than 1.  
  [Patterns, Functions, and Algebra Goal 1]

### Key Activities
Students review conversions from mixed numbers to fractions and from fractions to mixed numbers. Then they multiply mixed numbers by applying the conversions and by using the partial-products method.

### Ongoing Assessment:
- **Informing Instruction**  
  See page 661.

### Ongoing Assessment: Recognizing Student Achievement
- Use an Exit Slip (Math Masters, p. 414).  
  [Operations and Computation Goal 5]

### Materials
- Math Journal 2, pp. 272–274B
- Student Reference Book, pp. 77–78B
- Study Link 8•7
- Math Masters, p. 414

### Advance Preparation
For Part 1, draw several blank “What’s My Rule?” rule boxes and tables on the board to use with the Study Link 8•7 Follow-Up.

**Teacher’s Reference Manual, Grades 4–6** pp. 143, 144

### Using Unit Fractions to Find a Fraction of a Number
- **Math Journal 2**, p. 275
  Students practice using unit fractions to find a fraction of a number.

**Math Boxes 8•8**
- **Math Journal 2**, p. 276
  Students practice and maintain skills through Math Box problems.

**Study Link 8•8**
- **Math Masters**, p. 237
  Students practice and maintain skills through Study Link activities.

**READINESS**

**Ordering Improper Fractions**
- **slate**  
  Students review converting between fractions and mixed numbers and finding common denominators by ordering a set of improper fractions.

**EXTRA PRACTICE**

**Playing Frac-Tac-Toe**
- **Student Reference Book**, pp. 309–311
- **Math Masters**, pp. 472–484
  [per partnership: 4 each of number cards 0–10 (from the Everything Math Deck, if available), counters, calculator (optional)]
  Students practice converting between fractions, decimals, and percents.
Getting Started

**Mental Math and Reflexes**
Have students write each mixed number as a fraction. Suggestions:

- 1 $\frac{2}{3}$ $\frac{5}{10}$
- 2 $\frac{7}{7}$ $\frac{67}{67}$
- 3 $\frac{4}{5}$ $\frac{56}{8}$
- 4 $\frac{34}{9}$ $\frac{42}{5}$
- 5 $\frac{9}{9}$ $\frac{10}{5}$

**Math Message**
Complete journal page 272.

**Study Link 8-7 Follow-Up**
Have partners compare answers and resolve differences. Ask volunteers to write the incomplete version of their “What’s My Rule?” table for the class to solve.

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1 Teaching the Lesson

**Math Message Follow-Up**
(Math Journal 2, p. 272)

Ask students why the hexagons in the last row of the example on the journal page are divided into sixths. Sample answer: To add the 3 in 3 $\frac{5}{6}$, you need a common denominator. A simple way is to think of each whole as $\frac{6}{6}$. Ask volunteers to share their solution strategies for Problems 1–8.

**Multiplying with Mixed Numbers**
(Student Reference Book, pp. 77 and 78)

Ask students how they would use the partial-products method to calculate $6 \times 4 \frac{3}{5}$. Discuss student responses as you summarize the following strategy:

- Calculate the partial products and add.
  1. Think of $4 \frac{3}{5}$ as $4 + \frac{3}{5}$
     \[6 \times 4 \frac{3}{5} = 6 \times (4 + \frac{3}{5})\]
  2. Write the problem as the sum of partial products.
     \[= (6 \times 4) + (6 \times \frac{3}{5})\]
  3. Calculate the partial products.
     \[= 24 + \frac{18}{5}\]
  4. Convert $\frac{18}{5}$ to a mixed number.
     \[= 24 + 3\frac{3}{5}\]
  5. Add.
     \[= 27\frac{3}{5}\]

Refer students to Step 2. Ask: What property is used to rewrite the problem as the sum of partial products? The Distributive Property Write problems on the board or a transparency and ask students to find the missing value using the Distributive Property. Suggestions:

- $4 \times 3\frac{1}{2} = (4 \times 3) + (4 \times \frac{1}{2})$
- $4 \times 3\frac{1}{2} = ? 14$
- $8 \times 2\frac{1}{4} = (8 \times n) + (8 \times \frac{1}{4})$
- $8 \times 2\frac{1}{4} = ? 18$

Ask students to refer to the example on **Student Reference Book**, page 78.
Ask students how they might use improper fractions to calculate $6 \times 4\frac{2}{3}$. Again, discuss student responses as you summarize the following strategy:

Convert whole numbers and mixed numbers to improper fractions.

1. Think of $6$ as $\frac{6}{1}$ and $4\frac{2}{3}$ as $\frac{14}{3}$.
2. Rewrite the problem as fraction multiplication $6 \times 4\frac{2}{3} = \frac{6}{1} \times \frac{14}{3}$.
3. Use a fraction multiplication algorithm.
5. Simplify the answer by converting $\frac{138}{5}$ to a mixed number.

Ask students to refer to the second example on Student Reference Book, page 77.

Ask students to suggest advantages and disadvantages for each method. Expect a variety of responses. Sample answers: The partial-products method lets you work with smaller numbers but has more calculations. The improper-fraction method lets you multiply fractions where one of the denominators will be one, but you have a larger number to divide to simplify the answer.

Ask students to work through two or three additional examples, using either of the above strategies or others of their own choosing. After each problem, ask volunteers to share their solution strategies. Suggestions:

- $4 \times 3 \frac{2}{5}$
- $2 \frac{1}{4} \times 3 \frac{1}{2}$
- $2 \frac{2}{3} \times 3$

Ask: What observations can you make regarding the factors in the three problems and the products? Sample answers: When you multiply a nonzero whole number by a fraction less than 1 (as in $4 \times \frac{3}{5} = 2\frac{2}{5}$), the product will be smaller than the whole number. When you multiply a nonzero number by a number greater than 1 (as in $2\frac{3}{5} \times 3 = 8$), the product will be greater than the given number.

Pose some number stories for students to solve. Suggestions:

- Three students are making necklaces in art class. Each necklace needs $4\frac{3}{5}$ feet of string. How much string is needed for the three students to each make one necklace? $12\frac{3}{5}$, or $13\frac{1}{5}$ ft
- The top of a rectangular pencil case has a width of $2\frac{3}{5}$ inches and a length of 8 inches. What is the area of the top? $16\frac{24}{5}$, or 19 sq in.
**Multiplying Fractions and Mixed Numbers**

(Math Journal 2, pp. 273 and 274)

Assign both journal pages. Encourage students to consider the numbers in each problem and then to use the method that is most efficient for that problem. Circulate and assist.

**Interpreting Multiplication as Resizing (Scaling)**

(Math Journal 2, p. 274A; Student Reference Book, pp. 78A and 78B)

Explain to students that when they make an enlarged or reduced copy of an image using a photocopy machine, they are resizing or scaling the image. Ask students to tell where they have used the word scale in everyday life. Sample responses: I weigh things on a scale; an axis on a graph has a scale; I put together scale models; a map is a scale drawing. Conclude by pointing out that although scale models and scale drawings often are smaller than the original, scaling refers to the process of reducing, enlarging, or maintaining the same size. Ask students to refer to the examples on Student Reference Book, pages 78A and 78B.

Pose the following situations to students. Have students tell if the situation involves enlarging, reducing, or making a copy that is the same size. Also have students explain their answers.

- **You make a copy that is \( \frac{3}{4} \) the size of the original. Reducing; Sample answer: \( \frac{3}{4} \) is less than 1, so the copy will be smaller than the original.
- **You make a copy that is \( 1\frac{1}{2} \) times the size of the original. Enlarging; Sample answer: The copy will be the same size plus a half size more, so the copy will be larger than the original.
- **You make a copy that is 100% of the original size. Same size; Sample answer: 100% means one whole, so the copy will be full size.
Explain that the enlargement or reduction of an image is measured with numbers called size-change, or scale, factors.

Work through Problems 1 and 2 on Math Journal 2, page 274A as a class. Be sure the following points are discussed (for positive numbers):

- When you multiply a given number by a number greater than 1, the product is greater than the original number.
- When you multiply a given number by a number less than 1, the product is less than the original number.
- When you multiply a given number by a number equal to 1, the product is equal to the original number.

Then assign the rest of the journal page. Circulate and assist. After students complete the journal page, write the following expressions on the board.

\[
\frac{3}{2} \times \frac{3}{4} \quad 2\frac{8}{10} \times \frac{3}{4} \quad \frac{9}{10} \times \frac{3}{4} \quad \frac{9}{100} \times \frac{3}{4} \quad \frac{3}{3} \times \frac{3}{4}
\]

Ask students to determine, without performing any calculations, which of these expressions would result in the largest product and which would result in the smallest product. Ask them to explain their answers. Sample answers: In each case, \(\frac{3}{4}\) is multiplied by a number. The largest number that \(\frac{3}{4}\) is multiplied by is \(\frac{3}{3}\), so \(\frac{3}{2} \times \frac{3}{4}\) would result in the largest product. The smallest number that \(\frac{3}{4}\) is multiplied by is \(\frac{9}{100}\), so \(\frac{9}{100} \times \frac{3}{4}\) would result in the smallest product. Ask: Without doing any calculations, compare \(\frac{3}{4}\) with the product of \(3\frac{1}{2} \times \frac{3}{4}\) and the product of \(\frac{9}{100} \times \frac{3}{4}\). Sample answers: \(3\frac{1}{2} \times \frac{3}{4}\) is \(3\frac{1}{2}\) times as large as \(\frac{9}{100} \times \frac{3}{4}\), which is much smaller than \(\frac{3}{4}\); it is \(\frac{9}{100}\) the size of \(\frac{3}{4}\).

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**Student Reference Book, p. 78A**

Example:

Show that the 10-inch-scale length of the conifer in the scale drawing is a \(\frac{1}{10}\) reduction of the actual length. The length of the conifer in the real world is 10 in. Write a number model that fits this problem.

\[
\frac{1}{10}x = 10
\]

The drawing is a \(\frac{1}{10}\) reduction of the actual length.

Example:

A desktop model car is a reduction of an actual car. Whole scale factor might be small.

\[
x = \frac{1}{100}x
\]

A reduction means multiplying by a scale factor less than 1, or the possible scale factor is \(\frac{1}{100}\).

In general, when you multiply a given number by a positive number less than 1, the product is less than the given number.

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**Student Reference Book, p. 78B**

Lesson 8-8
Fraction Problems

(Math Journal 2, p. 274B)

Students solve real-world number stories involving fractions. They use a visual model or write an open number model to help them solve each problem.

Using Unit Fractions to Find a Fraction of a Number

(Math Journal 2, p. 275)

Students practice using unit fractions to find the fraction of a number.

Math Boxes 8-8

(Math Journal 2, p. 276)

Mixed Practice Math Boxes in this lesson are paired with Math Boxes in Lesson 8-6. The skill in Problem 5 previews Unit 9 content.

Writing/Reasoning Have students write a response to the following: Explain how to use the division rule for finding equivalent fractions to solve Problem 4b. Sample answer: The division rule states that you can rename a fraction by dividing the numerator and the denominator by the same nonzero number. I divided the numerator and the denominator by 2 to rename the fraction $\frac{4}{5} \div \frac{2}{5} = \frac{2}{25}$.

Study Link 8-8

(Math Masters, p. 237)

Home Connection Students practice multiplying fractions and mixed numbers. They find the areas of rectangles, triangles, and parallelograms.
3 Differentiation Options

READINESS

Ordering Improper Fractions

To review converting between fractions and mixed numbers and finding common denominators, have students order a set of improper fractions. Write the following fractions on the board: 7/2, 11/6, and 11/5. Ask students to suggest how to order the fractions from least to greatest. Expect that students will suggest the same strategies they used with proper fractions, such as putting the numbers in order and then comparing them to a reference. Use their responses to discuss and demonstrate the following methods:

 Rename each improper fraction as an equivalent fraction with a common denominator. Ask students what common denominator to use. Sample answer: Use 6 because the other denominators are all factors of 6. Have volunteers rename the fractions and write the equivalent fractions on the board underneath the first list of fractions. \[ \frac{21}{6}, \frac{24}{6}, \frac{14}{6}, \frac{11}{6} \]

 Write each fraction as a whole or mixed number. Ask volunteers to write these mixed numbers on the board underneath the second list. 3\( \frac{1}{2} \), 4, 2\( \frac{1}{2} \), 1\( \frac{5}{6} \)

Ask students to order the 3 lists on their slates. 11/6, 7/3, 7/4, 11/6, 14/6, 21/6, 24/6; and 11/6, 2\( \frac{1}{2} \), 3\( \frac{1}{2} \), 4 Discuss any difficulties or curiosities that students encountered.

EXTRA PRACTICE

Playing Frac-Tac-Toe

(Student Reference Book, pp. 309–311; Math Masters, pp. 472–484)

Students play a favorite version of Frac-Tac-Toe to practice converting between fractions, decimals, and percents.