Objectives
To introduce finding a fraction of a fraction.

Key Concepts and Skills
- Use unit fractions to find a fraction of a number and to find the whole. [Operations and Computation Goal 5]
- Use an area model to find fractions of fractions. [Operations and Computation Goal 5]

Key Activities
Students solve fraction-of problems using number lines and folded-paper manipulatives.

Ongoing Assessment: Recognizing Student Achievement
Use journal page 259. [Operations and Computation Goal 5]

Key Vocabulary
vertical • horizontal

Materials
Math Journal 2, pp. 259–261
Student Reference Book, pp. 74 and 75
Math Masters, p. 414
Study Link 8-4
transparency of Math Masters, p. 434 (optional) • 5 sheets of 8 1/2” by 11” paper per student • slate • scissors • poster board • Class Data Pad (optional)

Advance Preparation
For Part 1, prepare a labeled diagram illustrating 3/4 of 2/3 on a poster or the Class Data Pad. (See page 646.) Make a transparency of Math Masters, page 434. Have extra sheets of 8 1/2” by 11” paper available to students. For the Part 2 Adjusting the Activity, students will need 3” by 5” index cards cut in half vertically.


Playing Fraction Spin
Math Journal 2, p. 262
Math Masters, p. 471
per partnership: large paper clip, 15 index cards (optional)
Students practice estimating sums and differences of fractions.

Math Boxes 8-5
Math Journal 2, p. 263
Students practice and maintain skills through Math Box problems.

Study Link 8-5
Math Masters, p. 229
Students practice and maintain skills through Study Link activities.

Modeling Equivalent Fractions
Math Masters, p. 230
Students use a fraction stick to model equivalent fractions.

Summing the Squares
calculator • Class Data Pad (optional)
Students use their calculators to find sums of fractions.

Building a Math Word Bank
Differentiation Handbook, p. 142
Students illustrate and write words related to the terms horizontal and vertical.
Getting Started

Math Message Follow-Up
(Math Journal 2, p. 259; Math Masters, p. 434)

Ask volunteers to present their solution strategies for Problems 1–10 on the board or on a transparency of Math Masters, page 434. Expect student approaches to be similar to the following:

► Use the number line. For Problem 3, divide the segment from 0 to 2 into 4 equal parts, and count 3 of those parts starting from 0.

\[
\begin{align*}
0 & \quad 1 & \quad 2 & \quad 3 \\
\hline
\frac{3}{4} \text{ of 2 is } 1 & \frac{1}{2}
\end{align*}
\]

► For Problem 7, divide the segment from 0 to \(\frac{3}{4}\) into 2 equal parts, and count 1 of those parts.

\[
\begin{align*}
0 & \quad 1 & \quad 2 & \quad 3 \\
\hline
\frac{1}{2} \text{ of } \frac{3}{4} \text{ is } & \frac{3}{8}
\end{align*}
\]

► Use addition. For Problem 3, add \(\frac{3}{4}\) of 1 and \(\frac{3}{4}\) of 1 to find \(\frac{3}{4}\) of 2: \(\frac{3}{4} + \frac{3}{4} = \frac{6}{4} = 1 \frac{1}{2}\).

Ongoing Assessment: Recognizing Student Achievement

Use journal page 259, Problems 1–6 to assess students’ understanding of fractional parts on a number line. Students are making adequate progress if they correctly answer Problems 1–6. Some students might correctly answer Problems 7–10.

[Operations and Computation Goal 5]
Using Unit Fractions to Find a Fraction of a Number
(Math Journal 2, p. 259; Student Reference Book, pp. 74 and 75)

As a class, work through the examples in the Student Reference Book, pages 74 and 75. Then have students complete Problems 12–21 on journal page 259. Circulate and assist.

Modeling How to Find a Fraction of a Fraction

Tell students that they are going to fold paper to solve problems that ask for a fraction of a fraction.

Example 1: Larry has \( \frac{1}{2} \) of a fruit bar and wants to give half of it to his brother. How much of the whole fruit bar will Larry give to his brother? This problem can be solved by thinking of \( \frac{1}{2} \) of \( \frac{1}{2} \).

Ask students each to fold a sheet of paper into halves, either horizontally or vertically. Define vertical as the longer dimension of the paper and horizontal as the shorter dimension. To support English language learners, write horizontal and vertical on the board along with examples and discuss their meanings. Compare their results. It doesn’t matter which way students fold their sheets in this example, but the way they fold them will be important in the next example.

Have students unfold their sheets, and show them how to orient the sheets so the halves are to the left and right of each other (rather than above and below each other). Shade the left half with diagonal marks. (See margin.)

Ask the students to fold their papers into halves in the opposite direction. (If the first fold was horizontal, the second fold will be vertical, and vice versa.) Have students unfold their sheets, orient them so the new halves are above and below each other, and shade the bottom half with diagonal marks that slant in the opposite direction to the first shading. (See margin.)

Ask a volunteer to give the answer and explain how this folded-paper model shows how much of the fruit bar Larry gave to his brother. Remind students that the sheet of paper represents the whole fruit bar, or ONE. Sample answer: Larry gave his brother \( \frac{1}{4} \) of the fruit bar. Folding and shading the paper to represent \( \frac{1}{2} \) of \( \frac{1}{2} \) divided the whole into 4 equal parts. One of those 4 equal parts was shaded twice. This shows that \( \frac{1}{2} \) of \( \frac{1}{2} \) is \( \frac{1}{4} \).

Students write \( \frac{1}{2} \) of \( \frac{1}{2} \) is \( \frac{1}{4} \) on their folded papers.
Example 2: Ava has $\frac{1}{2}$ of a pizza. She eats $\frac{2}{3}$ of the pizza she has. How much of the whole pizza did she eat? This problem can be solved by thinking $\frac{2}{3}$ of $\frac{1}{2}$.

Ask students how they would fold and shade a sheet of paper to find $\frac{2}{3}$ of $\frac{1}{2}$. Refer to their suggestions as you lead students through the following steps:

1. Note that the denominator of the second fraction is 2. Fold a sheet of paper into halves, vertically. Unfold it and shade the left half.

2. Note that the denominator of the first fraction is 3. Fold the sheet approximately into thirds, horizontally.

3. Unfold the sheet and use different shading to shade the bottom $\frac{2}{3}$ of the sheet, using the new folds. Explain that you are shading $\frac{2}{3}$ of the entire sheet and also $\frac{2}{3}$ of each half.

4. Write an X in each of the two parts of the sheet that was shaded twice. These parts represent $\frac{2}{3}$ of $\frac{1}{2}$.

Math Journal 2, p. 261

Adjusting the Activity

Have students find $\frac{1}{2}$ of $\frac{2}{3}$. Compare the answer and the folded-paper model to the model for $\frac{2}{3}$ of $\frac{1}{2}$. Students should conclude that $\frac{1}{2}$ of $\frac{2}{3}$ and $\frac{2}{3}$ of $\frac{1}{2}$ have the same answer.

Links to the Future

The paper-folding model anticipates the area model that is developed in Lesson 8-6. Both models are used to prepare students for multiplying fractions.
Ask a volunteer to give the answer and explain how this folded-paper model shows how much of the whole pizza Ava ate. Remind students that the sheet of paper represents the whole pizza, or ONE. Sample answer: Ava ate \( \frac{2}{3} \), or \( \frac{1}{3} \), of the pizza. Folding and shading the paper to represent \( \frac{2}{3} \) of \( \frac{1}{3} \) divided the whole into 6 equal parts. Two of those 6 equal parts were shaded twice. This shows that \( \frac{2}{3} \) of \( \frac{1}{3} \) is \( \frac{2}{6} \).

Ask: How does the product \( \frac{3}{4} \) compare to the two factors \( \frac{3}{4} \) and \( \frac{1}{2} \)? Sample answer: The product is smaller than both of the factors; the product \( \frac{3}{4} \) is \( \frac{3}{8} \) as much as \( \frac{1}{2} \).

Have students write \( \frac{3}{4} \) of \( \frac{1}{3} \) is \( \frac{2}{6} \) on their sheets.

**Finding a Fraction of a Fraction**

(Math Journal 2, pp. 260 and 261; Math Masters, p. 414)

Have students fold paper to find \( \frac{1}{3} \) of \( \frac{3}{4} \) and \( \frac{3}{4} \) of \( \frac{1}{3} \). Circulate and assist. When most students have finished, ask volunteers to demonstrate their folded-paper model solutions.

Present the labeled diagram illustrating \( \frac{3}{4} \) of \( \frac{1}{3} \) that you prepared. Emphasize that the whole can be thought of as fourths, thirds, or twelfths, but the parts that are shaded twice represent twelfths.

![Diagram](image)

Have students fold and shade a sheet of paper to show \( \frac{3}{4} \) of \( \frac{2}{3} \), and then cut out the rectangular pieces representing twelfths. Ask a volunteer if the answer on the diagram is in simplest form. No

Ask students to rearrange their pieces so they show the answer in simplest form. The answer in simplest form is \( \frac{1}{2} \). Pieces should be arranged in 3 rows and 4 columns so that the pieces shaded twice represent \( \frac{1}{2} \) because \( \frac{6}{12} = \frac{3}{6} = \frac{1}{2} \).

Ask students to complete journal pages 260 and 261. Tell them to think of the rectangles as sheets of paper and to sketch the folds and shading. They should estimate where the folds are on each sketch and then fill in their answers.

After completing Math Journal 2, pages 260 and 261, have students write a number story that matches Problem 4, Math Journal 2, page 260 (\( \frac{3}{4} \) of \( \frac{1}{2} \)) on an Exit Slip (Math Masters, page 414).
**2 Ongoing Learning & Practice**

**Playing Fraction Spin**

(Math Journal 2, p. 262; Math Masters, p. 471)

Students play *Fraction Spin* to practice estimating sums and differences of fractions. Students will recognize *Fraction Spin* as being the same as *Mixed-Number Spin* from Lesson 8-3. This version has fractions only. Remind students to use benchmarks as they make their estimates.

**Adjusting the Activity**

Have students make a deck of 30 cards with fractions of their choice and a record sheet to match. Instead of using the spinner, partners take turns drawing cards and completing their record page.

To make a deck, partners need to decide on a range for their cards. They each make 15 cards for the deck. They can make the cards on 3 in. by 5 in. cards, cut in half.

**Math Boxes 8-5**

(Math Journal 2, p. 263)

**Mixed Practice** Math Boxes in this lesson are paired with Math Boxes in Lesson 8-7. The skills in Problems 4 and 5 preview Unit 9 content.

**Writing/Reasoning** Have students write a response to the following: Write one of the false number sentences from Problem 2. Then write it correctly so it is true, and explain your solution. Sample answer: 16 – (3 + 5) = 18 is false because 16 – 3 + 5 = 18. Remove the parentheses and follow the order of operations, 16 – 3 + 5 = 18.

**Study Link 8-5**

(Math Masters, p. 229)

**Home Connection** Students solve fraction-of-a-fraction problems.
**3 Differentiation Options**

**READINESS**

▲ **Modeling Equivalent Fractions**

* (Math Masters, p. 230)

To reinforce students’ understanding of modeling fractional parts, work through the Math Masters page as a group. Discuss questions such as the following:

- When using a fraction stick, how do you know that two fractions are equivalent? *The pieces are the same size.*
- How could you express the whole as a fraction with a denominator of 8? \( \frac{8}{8} \)
- Explain how you can tell that a fraction represents 1 or the whole. *The numerator and the denominator are the same.*

**ENRICHMENT**

▲ **Summing the Squares**

To extend students’ ease in performing operations with fractions using a calculator, have them explore finding sums of fractions. Ask students for two numbers whose sum is 1. List these addition expressions on the board or Class Data Pad. Tell students to review the list, and ask: *If one of the fractions is squared, which sum is larger: the sum of the larger fraction squared and the smaller fraction, or the sum of the smaller fraction squared and the larger fraction?* Ask one group to find the sum after squaring the larger number and another group to find the sum after squaring the smaller number. Have a student from each group list the number sentences on the board.

*For example:* \( \left( \frac{3}{4} \right)^2 + \frac{3}{8} = \frac{13}{16} \), and \( \frac{3}{4} + \left( \frac{5}{8} \right)^2 = \frac{52}{64} = \frac{13}{16} \).

When students have completed the lists, discuss any problems or curiosities they encountered, and then discuss their findings. *The sums are the same.*

**ELL SUPPORT**

▲ **Building a Math Word Bank**

* (Differentiation Handbook, p. 142)

To provide language support for finding a fraction of a fraction, have students use the Word Bank Template found on Differentiation Handbook, page 142. Ask students to write the terms *horizontal* and *vertical*, draw pictures relating to each term, and write other related words. See the Differentiation Handbook for more information.