Subtracting Mixed Numbers

Objective: To develop subtraction concepts related to mixed numbers.

**Key Concepts and Skills**
- Find equivalent names for mixed numbers.  
  [Number and Numeration Goal 5]
- Convert between fractions and mixed numbers.  
  [Number and Numeration Goal 5]
- Subtract mixed numbers.  
  [Operations and Computation Goal 4]
- Use benchmarks to estimate differences.  
  [Operations and Computation Goal 6]

**Key Activities**
Students subtract mixed numbers with like denominators by renaming the minuend. Students use benchmarks to estimate differences.

**Materials**
- Math Journal 2, p. 254
- Math Masters, p. 414
- Study Link 8-2 slate

**Advance Preparation**
- Teacher’s Reference Manual, Grades 4–6 pp. 142, 143

**Differentiation Options**

**Readiness**
Subtracting Mixed Numbers
Students use a counting-up algorithm to explore mixed-number subtraction.

**Enrichment**
Exploring a Pattern for Fraction Addition and Subtraction
Math Masters, p. 225
Students explore patterns that result from adding and subtracting unit fractions.

**Extra Practice**
5-Minute Math
5-Minute Math™, pp. 184 and 185
Students add mixed numbers.
Getting Started

Mental Math and Reflexes
Have students name a common denominator for the following:
- halves and thirds, sixths
- fifths and thirds, fifteenths
- fourths and eighths, eighths
- fifths and eighths, fortieths
- halves, thirds, and fourths, twelfths
- halves, fifths, and sixths, thirtieths

Math Message
Solve Problems 1–3 at the top of journal page 254.
Use benchmarks to estimate the differences and to check the reasonableness of your solutions.

Study Link 8-2 Follow-Up
Have partners compare answers and resolve differences. Ask volunteers how they know their answers are in simplest form. A fraction is in simplest form if the numerator and denominator have no common factors except 1.

1 Teaching the Lesson

Math Message Follow-Up
(Math Journal 2, p. 254)

Ask students to share their estimating strategies for Problem 2 using benchmarks. Sample answer: I know that 4 \(\frac{3}{5}\) is close to 5 and 5 – 2 = 3. I estimated my answer to be about 3.

Ask volunteers to write their solutions on the board and explain their strategies. Expect that most students will have subtracted the whole-number and fraction parts separately and renamed the differences in Problems 1 and 3 in simplest form.

Subtracting Mixed Numbers with Renaming

Write this problem on the board.

\[
3 \frac{1}{3} - 1 \frac{2}{3}
\]

Ask: How does this problem differ from the Math Message problems? The fraction being subtracted from, \(\frac{1}{3}\), is smaller than the fraction that is being subtracted, \(2 \frac{2}{3}\). Ask volunteers to suggest solution strategies. Sample answer: Rename \(3 \frac{1}{3}\) to make the fraction part larger. \(2 \frac{2}{3} - 1 \frac{2}{3} = 1 \frac{1}{3}\)

Remind students that when they add mixed numbers, some sums might have to be renamed to write them in simplest form. For example, a sum of \(4 \frac{3}{4}\) could be renamed: \(4 \frac{3}{4} = 4 + \frac{3}{4} + \frac{3}{4}\), or \(4 + 1 + \frac{3}{4} = 5 \frac{3}{4}\). Emphasize that the two mixed numbers are equivalent; \(4 \frac{3}{4}\) is another name for \(5 \frac{3}{4}\).

Student Page

Subtracting Mixed Numbers

Subtract.

\[
6 \frac{1}{5} - 3 \frac{1}{5}
\]

Renaming and Subtracting Mixed Numbers

Fill in the missing numbers.

\[
4 \frac{1}{2} - 3 \frac{1}{2}\]

Subtract. Write your answers in simplest form. Show your work.

\[
7 \frac{1}{4} - 4 \frac{1}{4}
\]

How many more hours does it take to practice this week? So far, he has practiced \(4 \frac{1}{2}\) hours. How many more hours does he need to practice this week?
Ongoing Assessment:
Informing Instruction
Watch for students who have difficulty renaming mixed numbers for subtraction.
Have students practice the following steps:
1. Write the whole-number part of the mixed number as an equivalent addition expression that adds 1.
   \[ \frac{5}{2} = 4 + \frac{2}{5} \]
2. Use the denominator to rename the 1 as a fraction.
   \[ \frac{5}{2} = 4 + \frac{2}{5} \]
3. Combine the fraction parts.
   \[ \frac{5}{2} = 4 + \frac{2}{5} \]

Have students rename \(\frac{3\frac{1}{3}}{\frac{3}{5}}\) to an equivalent mixed number with a larger fraction part. \(\frac{1\frac{1}{3}}{\frac{2\frac{3}{5}}{2}}\) Write their responses on the board or a transparency, and illustrate the mixed numbers with pictures.

Ask students to use each of the responses to solve the problem. Circulate and assist. When most students have finished, have volunteers explain which mixed-number name was more efficient for solving the subtraction problem. \(\frac{2\frac{3}{4}}{\frac{1\frac{1}{3}}{2}}\) because with \(\frac{1\frac{1}{3}}{2}\) you need to rename the fraction again in order to express the difference as a mixed number. Explain that deciding how to rename a fraction depends on the problem. Sometimes it might be more efficient to work with mixed numbers, other times it might be more efficient to work with improper fractions. In this case, the more efficient mixed-number name is a matter of personal preference.

Pose several problems. Encourage students to estimate each difference using what they know about benchmarks with fractions. They should use their estimates to check the reasonableness of their solutions. In each case, ask students first how they would rename the minuend and then how they would solve the problem.

Suggestions:
- \(8 - \frac{3\frac{2}{3}}{\frac{8}} = 7\frac{3}{8}\); solution: \(4\frac{1}{3}\)
- \(6 - \frac{\frac{1}{4}}{\frac{6}} = 5\frac{3}{4}\); solution: \(5\frac{3}{4}\)
- \(\frac{4\frac{5}{8}}{\frac{14}{5}} - \frac{\frac{4}{3}}{\frac{5}{2}} = \frac{3\frac{8}{5}}{\frac{4}{5}}\); solution: \(2\frac{4}{5}\)
- \(\frac{5\frac{1}{3}}{\frac{6}} - \frac{\frac{2\frac{1}{6}}{6}}{\frac{1}{6}} = \frac{4\frac{7}{6}}{\frac{2\frac{1}{3}}{6}}\); solution: \(2\frac{1}{3}\)
- \(\frac{6\frac{5}{12}}{\frac{3\frac{11}{12}}{6}} - \frac{\frac{\frac{5}{5}}{\frac{12}}}{\frac{12}} = \frac{\frac{5\frac{17}{12}}{\frac{12}}}{\frac{2\frac{1}{2}}{12}}\); solution: \(2\frac{1}{2}\)
- Carlie purchased 10 feet of ribbon to make bows. She used \(\frac{2\frac{3}{5}}{\frac{3}}\) feet to make one bow. How much ribbon remains to make additional bows? \(7\frac{1}{8}\) feet
- Sammy needs \(3\frac{1}{4}\) cups of sugar to make sugar cookies. He only has \(1\frac{3}{4}\) cups of sugar. How much more sugar does he need for the recipe? \(1\frac{1}{2}\) cups
Write this problem on the board.

\[ \frac{3}{1\frac{3}{2}} - \frac{1\frac{1}{2}}{\phantom{1\frac{3}{2}}} \]

Ask: How would you solve this problem? Expect that students will likely recognize \( \frac{1}{3} \) is smaller than \( \frac{1}{2} \) and might suggest renaming the fraction parts with common denominators. Ask volunteers to model their strategies on the board. Sample answer: Use 6 as a common denominator. Rename the mixed numbers. \( \frac{3}{2} - \frac{1\frac{1}{2}}{\phantom{1\frac{3}{2}}} = \frac{3}{6} - \frac{1\frac{3}{6}}{\phantom{1\frac{3}{6}}} \). Subtract. \( \frac{2\frac{5}{6}}{\phantom{1\frac{3}{6}}} - \frac{1\frac{3}{6}}{\phantom{1\frac{3}{6}}} = \frac{5}{6} \).

Pose a few mixed-number subtraction number stories. Suggestions:

- Daniel has a DVD set that provides 4\( \frac{1}{2} \) hours of viewing. So far, he has watched \( \frac{3}{4} \) of an hour. How many more hours of the DVD set does he still have to watch? \( 3\frac{4}{6} \) or \( 3\frac{3}{2} \) hours.
- Serena measured out 5\( \frac{2}{3} \) feet of rope to make a jump rope. Her friends told her that it needed to be 6\( \frac{3}{4} \) feet long. How much more rope did she need to measure out? \( \frac{5}{4} \) ft more.

**Subtracting Mixed Numbers**

*(Math Journal 2, p. 254; Math Masters, p. 414)*

Ask students to complete the journal page. Remind them to use benchmarks to estimate the difference. Ask students to use an Exit Slip *(Math Masters, page 414)* to explain their strategy for solving Problem 10 on *Math Journal 2*, page 254.

**Ongoing Learning & Practice**

**Playing Mixed-Number Spin**

*(Math Journal 2, p. 255; Math Masters, pp. 488 and 489)*

**Algebraic Thinking** Students practice estimating sums and differences of mixed numbers with like and unlike denominators by playing *Mixed-Number Spin*. Players use benchmarks to estimate sums and differences as they record number expressions that fit the parameters given on the *Mixed-Number Spin* record sheet, *Math Journal 2*, page 255. Each partnership makes a spinner using a large paper clip anchored by a pencil point to the center of the spinner on *Math Masters*, page 488. They use the numbers they spin to complete the number sentences on the record sheet.

Ask students to save their spinners for future use. Copy *Math Masters*, page 489 to provide additional record sheets.
Math Boxes 8-3

(Math Journal 2, p. 256)

Mixed Practice Math Boxes in this lesson are paired with Math Boxes in Lesson 8-1. The skill in Problem 5 previews Unit 9 content.

Writing/Reasoning Have students write a response to the following: Explain one advantage and one disadvantage to using number and word notation in Problem 2. Sample answer: One advantage is that the number is written the same way that it is said aloud. One disadvantage is that you cannot perform operations on them as easily.

Writing/Reasoning Have students write a response to the following: Explain how you determined your answer to Problem 3c. Sample answer: The answer is true because a parallelogram has two pairs of parallel sides with opposite sides congruent. When all sides of a parallelogram are the same length and form right angles, the parallelogram is also a square.

Study Link 8-3

(Math Masters, p. 224)

Home Connection Students practice renaming and subtracting mixed numbers.

3 Differentiation Options

Subtracting Mixed Numbers

To explore mixed-number subtraction, have students use a counting-up algorithm. Remind students that another way to subtract is to count up. With whole numbers, they could solve 5 – 3 by thinking 3 more what equals 5? Discuss how they would count up to solve this problem: \(\frac{5}{3} - \frac{3}{5}\). Expect that students will suggest “3\(\frac{4}{5}\)” plus what equals \(5\frac{2}{5}\). Explain that counting up is one way to answer plus what?

Demonstrate how to count up from \(3\frac{4}{5}\) to \(5\frac{2}{5}\).

1. Count up, keeping track of the amounts used.
   - Count up \(\frac{1}{5}\): \(3\frac{4}{5} + \frac{1}{5} = 4\frac{1}{5}\)
   - Count up 1: \(4 + 1 = 5\)
   - Count up \(\frac{2}{5}\): \(5 + \frac{2}{5} = 5\frac{2}{5}\)

2. Add \(\frac{1}{5} + \frac{2}{5} = 1\frac{3}{5}\).

So the difference between \(5\frac{2}{5}\) and \(3\frac{4}{5}\) is \(1\frac{3}{5}\).
Pose problems for students to solve by counting up. **Suggestions:**

- 4 5/8 – 1 7/8 2 3/8
- 3 5/8 – 1 4/8 1 4/8
- 7 – 2 8/10 4 1/5
- 5 1/4 – 2 5/4 2 1/2

**ENRICHMENT**

**Exploring a Pattern for Fraction Addition and Subtraction**

(*Math Masters, p. 225*)

**Algebraic Thinking** To apply students’ understanding of addition and subtraction of fractions, have them find patterns in unit fraction addition and subtraction problems. When most students have completed the problems, discuss the pattern in Problem 1. In each problem, the answer is the sum of the denominators over the product of the denominators.

Problem 4 asks whether this pattern works for the sum of any two unit fractions 1/a + 1/b. It does. Add 1/3 and 1/5 using the QCD method:

\[
\frac{1}{3} + \frac{1}{5} = \frac{1 \cdot 5 + 1 \cdot 3}{3 \cdot 5} = \frac{5 + 3}{15} = \frac{8}{15}
\]

Have students verify that this pattern holds true for other unit fractions, for example: \(\frac{1}{4} + \frac{1}{3} = \frac{1}{12}\).

The pattern for Problem 5 is similar. The denominator of the result is the product of the denominators of the unit fractions being subtracted. The numerator is always 1.

Problems 5a–5e involve the subtraction of two unit fractions where the difference between the denominators is 1. The pattern described in Problem 5f reflects that type of problem. You may want to extend the **Math Master** activity by having students find the results of the following subtraction problems (where the difference between the denominators of the unit fractions is greater than 1):

\[
\frac{1}{2} - \frac{1}{5} = \frac{3}{10} \quad \frac{1}{3} - \frac{1}{8} = \frac{5}{24} \quad \frac{1}{5} - \frac{1}{12} = \frac{7}{60}
\]

Ask students to describe a pattern for subtracting any two unit fractions. **Sample answer:** The numerator of the result is found by subtracting the denominator of the first fraction from the denominator of the second fraction. The denominator of the result is the product of the denominators of the two unit fractions.

**EXTRA PRACTICE**

**5-Minute Math**

To offer students more experience with adding mixed numbers, see **5-Minute Math**, pages 184 and 185.