Objective To introduce division of fractions and relate the operation of division to multiplication.

Key Concepts and Skills
• Find common denominators for pairs of fractions. [Number and Numeration Goal 5]
• Use diagrams and visual models for division of fractions problems. [Operations and Computation Goal 5]
• Solve number stories involving division of a fraction by a whole number, division of a whole number by a fraction, and division of a fraction by a fraction. [Operations and Computation Goal 5]
• Write equations to model number stories. [Patterns, Functions, and Algebra Goal 2]

Key Activities
Students use diagrams and visual models to divide fractions. They solve number stories involving division of a fraction by a whole number, division of a whole number by a fraction, and division of a fraction by a fraction. Students use visual fraction models and equations to represent the problem.

Ongoing Assessment: Informing Instruction
See page 683.

Ongoing Assessment: Recognizing Student Achievement
Use journal page 289. [Operations and Computation Goal 5]

Materials
Math Journal 2, pp. 288–289B
transparency of Math Masters, p. 440B
Student Reference Book, pp. 79–80B
Study Link 8-11
slate or half-sheets of paper

Math Boxes 8-12
Math Journal 2, p. 290
Students practice and maintain skills through Math Box problems.

Study Link 8-12
Math Masters, p. 248
Students practice and maintain skills through Study Link activities.

Enrichment
Exploring the Meaning of the Reciprocal
Math Masters, p. 249
calculator
Students explore the meaning of the reciprocal.

Extra Practice
Dividing with Unit Fractions
Math Masters, p. 253B
Students practice using visual models to divide fractions.

Advance Preparation
Math Message
Solve Problems 1 and 2 on journal page 288.

Study Link 8-11 Follow-Up
Have partners share answers and resolve differences. Ask volunteers to explain their solution strategies for Problem 9.

1. a. How many 2s are in 6?  
   b. How many \( \frac{1}{2} \)s are in 6?  
   c. How many \( \frac{1}{8} \)s are in \( \frac{3}{4} \)?  

2. a. How many 2s are in 10?  
   b. How many \( \frac{1}{2} \)s are in 10?
Refer students to the illustrations and questions for Problems 1 and 2, and ask what division open number sentence fits the first question. \(6 \div 2 = s\)

Continue for the other problems, writing the division open number sentence next to each question on the board. Ask students to refer to the visual models, if needed.

1. a. How many 2s are in 6? \(6 \div 2 = s\)
   b. How many \(\frac{1}{2}\)s are in 6? \(6 \div \frac{1}{2} = s\)
   c. How many \(\frac{1}{8}\)s are in \(\frac{3}{4}\)? \(\frac{3}{4} \div \frac{1}{8} = s\)

2. a. How many 2s are in 10? \(10 \div 2 = b\)
   b. How many \(\frac{1}{2}\)s are in 10? \(10 \div \frac{1}{2} = b\)

**NOTE** The division number models use \(b\) (for the number of boxes) and \(s\) (for the number of segments) to represent the unknowns. Students may prefer to use other letters or symbols. To avoid confusion in this introduction to division of fractions, the number models use ÷ rather than / to show division.

**Dividing with Unit Fractions**

(Math Journal 2, p. 289; Student Reference Book, pp. 79 and 80A)

Read and discuss the first example on page 79 of the Student Reference Book on dividing a whole number by a unit fraction. A unit fraction is a fraction with a numerator of 1. Briefly discuss the solution.

- Draw 3 rectangles on the board, and ask students to copy the rectangles on a sheet of paper or slate.

- Ask students to use the rectangles to illustrate the following problem: Jane has 3 loaves of banana bread to share with her friends. If she cuts each loaf into \(\frac{1}{4}\)s, how many quarter loaves will she have to share with her friends?

Students should conclude that one way to illustrate the solution is to divide each of the rectangles into 4 equal parts.

\[
\begin{array}{ccc}
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\end{array}
\]

\[3 \div \frac{1}{4} = 12\]

Ask:

- How many \(\frac{1}{4}\)s are in 3? 12
- What number model represents this problem? \(3 \div \frac{1}{4} = 12\)

Ask students to write \(3 \div \frac{1}{4} = 12\) below their rectangles.
Lesson 8-12

A unit fraction can also be divided by a whole number. Read and discuss the examples on page 80A of the Student Reference Book.

Draw a rectangle on the board, and divide it into 5 equal parts with \(\frac{1}{5}\) shaded. Ask students to draw the same diagram on a piece of paper or slate.

Tell students that you can represent a unit fraction (such as \(\frac{1}{5}\)) being divided by a whole number (such as 3) by drawing a model for the fraction and then cutting it up into smaller equal parts. Pose the following problem: *Three family members equally share \(\frac{1}{5}\) of a loaf of corn bread. How much of the loaf of corn bread will each person get?*

Have students divide their rectangles to show how the corn bread can be divided to find the solution to the problem.

**Ongoing Assessment:**

Informing Instruction

Watch for students who record the answer as \(\frac{1}{3}\). Have students draw the line for thirds to extend across the rectangle in order to visualize the total number of parts out of 15.

Have a volunteer come to the board to show the solution. The student should divide the shaded fifth into three equal parts using horizontal lines. If necessary, model the lines extended all the way across the larger rectangle, with one small part double shaded. Explain that because the family only has \(\frac{1}{5}\) of a loaf to begin with, when it is divided into three equal parts, each part of the corn bread that is cut up is \(\frac{1}{15}\) of the entire loaf. So each person will get \(\frac{1}{15}\) of the loaf of corn bread.

\[
\frac{1}{5} \div 3 = \frac{1}{15}
\]

Ask: *What number model represents this problem?* \(\frac{1}{5} \div 3 = \frac{1}{15}\)

Ask students to write “\(\frac{1}{5} \div 3 = \frac{1}{15}\)” below their rectangles.
Have students read through Problems 1–4 on journal page 289. Ask them to describe how Problems 1 and 2 are different from Problems 3 and 4. Sample answer: In Problems 1 and 2, you are dividing a whole number by a unit fraction. In Problems 3 and 4, you are dividing a unit fraction by a whole number.

Have students solve Problems 1–6. Circulate and assist. Briefly discuss solutions.

**Ongoing Assessment:**
**Recognizing Student Achievement**

Use journal page 289, Problems 1 and 2 to assess students’ ability to divide a whole number by a unit fraction using a visual model. Students are making adequate progress if they are able to solve Problems 1 and 2. Some students may be able to solve Problems 3 and 4, which involve dividing a unit fraction by a whole number.

[Operations and Computation Goal 5]

### Relationship between Multiplication and Division

(Math Journal 2, p. 289A)

Another way to solve a fraction division problem is to think about it as a related fraction multiplication problem. Remind students of the relationship between multiplication and division. For example, to solve $\frac{63}{7}$, you can think: What number times 7 is 63? or $n \times 7 = 63$. $n = 9$

Write the following problems on the board to show how the relationship helps when dividing with fractions.

- To solve $\frac{1}{10} \div 5$, think: What number times 5 is $\frac{1}{10}$? Or $n \times 5 = \frac{1}{10}$.
- To solve $6 \div \frac{1}{5}$, think: What number times $\frac{1}{5}$ is 6? Or $n \times \frac{1}{5} = 6$. $n = 30$
- To solve $6 \div \frac{2}{3}$, think: What number times $\frac{2}{3}$ is 6? Or $n \times \frac{2}{3} = 6$. $n = 9$
- To solve $\frac{1}{10} \div \frac{3}{10}$, think: What number times $\frac{3}{10}$ is $\frac{1}{10}$? Or $n \times \frac{3}{10} = \frac{1}{10}$. $n = \frac{1}{3}$

Ask students to solve the problems on journal page 289A. Circulate and assist.

### Introducing Common Denominator Division

(Math Journal 2, p. 289B)

Draw four circles on the board, and ask students to copy these circles on a sheet of paper. Ask them to solve the problem $4 \div \frac{2}{3}$ and to illustrate their solution using the four circles.
After a few minutes, bring the class together to discuss their solutions. Use the students’ responses to emphasize that one method for obtaining the solution is to divide each of the circles into 3 equal parts. As you illustrate this method on the board, point out that dividing each circle into 3 equal parts is equivalent to renaming each whole as \( \frac{3}{3} \). This shows that 4 wholes is equivalent to \( \frac{12}{3} \).

Write \( \frac{12}{3} \div \frac{2}{3} = 6 \) under the circles on the board. Students will readily see that there are 6 groups of \( \frac{2}{3} \). Emphasize that the answer is the result of dividing the numerators \( 12 \div 2 = 6 \).

Guide the discussion toward the following algorithm for division with fractions:

**Step 1** Rename the numbers using a common denominator.

**Step 2** Divide the numerators, and divide the denominators.

Discuss the examples at the top of journal page 289B. Point out that this method works for fractions divided by fractions or for mixed numbers or whole numbers divided by fractions. Use the following example to show that this method also works for fractions divided by whole numbers.

\[
\frac{1}{6} \div 6 = \frac{1}{8} \div \frac{48}{8} = \frac{1 \times 6}{8 \times 6} = \frac{\cancel{48}}{\cancel{48}} = \frac{1}{1} = \frac{1}{6}
\]

Solve Problems 1–3 on journal page 289B as a class. Ask students to come up to the board to record their steps.

### Links to the Future

The algorithm introduced in this lesson focuses students on the meaning of division with fractions. The standard algorithm that involves multiplying by the reciprocal will be introduced in *Sixth Grade Everyday Mathematics.*
**Differentiation Options**

### Ongoing Learning & Practice

**Math Boxes 8.12**

(Math Journal 2, p. 290)

**Mixed Practice** Math Boxes in this lesson are paired with Math Boxes in Lesson 8-10. The skill in Problem 6 previews Unit 9 content.

**Study Link 8.12**

(Math Masters, p. 248)

**Home Connection** Students practice operations with fractions and mixed numbers.

### 3. Playing Build-It

(Student Reference Book, p. 300; Math Masters, pp. 446 and 447)

To practice comparing and ordering fractions and renaming mixed numbers as fractions, have students play this variation of Build-It. If students did not keep their Fraction Cards, they will need to cut from Math Masters, page 446.

Students play the game as introduced in Lesson 8-1 except that at the end of each round, they toss a six-sided die to determine a whole-number part for each of their 5 fractions. Students then rename the mixed numbers as fractions. For example, after tossing a 3, the fractions \(\frac{1}{5}, \frac{1}{4}, \frac{7}{12}, \) and \(\frac{5}{8}\) would become \(\frac{3}{5}, \frac{3}{4}, \frac{3}{3}, \frac{7}{12}\), and \(\frac{3}{5}\). Renamed as fractions, the list would be \(\frac{16}{5}, \frac{13}{4}, \frac{10}{3}, \frac{43}{12},\) and \(\frac{29}{6}\).
Exploring the Meaning of the Reciprocal
(Math Masters, p. 249)

To explore the relationship between a number and its reciprocal, have students use what they know about fractions, fraction multiplication, and their calculators to find the reciprocals of numbers.

When students have finished the Math Masters page, ask them to describe the pattern for finding the reciprocal of a number. Guide students to see that the reciprocal of a fraction is the fraction with the numerator and denominator interchanged, or inverted.

For example, the reciprocal of \( \frac{4}{9} \) is \( \frac{9}{4} \), and \( \frac{4}{9} \times \frac{9}{4} = \frac{36}{36} = 1 \). The reciprocal of a whole number is a unit fraction that has the whole number as its denominator. For example, the reciprocal of 8 is \( \frac{1}{8} \), so \( 8 \times \frac{1}{8} = \frac{8}{8} = 1 \).

Dividing with Unit Fractions
(Math Masters, p. 253B)

Students practice using visual models to divide fractions.

Example 1:
\[
\frac{3}{5} \times \frac{1}{3} = \frac{3}{15} = \frac{1}{5}
\]

Example 2:
\[
\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}
\]

2. Find the reciprocal.
   a. \( \frac{1}{2} \)  b. \( \frac{7}{8} \)  c. \( \frac{5}{8} \)  d. \( \frac{9}{10} \)

3. What do you think would be the reciprocal of \( \frac{1}{4} \)?

Reciprocals on a Calculator
On all scientific calculators, you can find a reciprocal of a number by raising the number to the \(-1\) power.

4. Write each number in standard notation as a decimal and a fraction.
   a. \( 0.125 \)  b. \( 0.04 \)  c. \( 3 \)  d. \( 0.125^{-1} \)

5. Write the key sequences you could use to find the reciprocal of 36.
   a. \( 36 \div \)  b. \( 36 \div \)  c. \( 36 \div \)

6. Write the key sequences you could use to find the reciprocal of \( \frac{1}{10} \).
   a. \( 1.0 \div \)  b. \( 1.0 \div \)  c. \( 1.0 \div \)

7. What pattern do you see for the reciprocal of a fraction?

8. Once the original number is written as a fraction, the reciprocal is the original fraction written with the numerator as the denominator and the denominator as the numerator.

Math Masters, p. 249
Mixed-Number Review

1. a. Four pizzas will each be cut into eighths. Show how they can be cut to find how many slices there will be in all.

   b. The drawing shows that $4 \div \frac{1}{8} = \underline{\text{______}}$, so there will be \underline{______} slices in all.

2. a. Two families equally share $\frac{1}{3}$ of a garden. Show how they can divide their portion of the garden.

   b. The drawing shows that $\frac{1}{3} \div 2 = \underline{\text{______}}$, so each family gets \underline{______} of the total garden.

Common Denominator Division

Step 1 Rename the numbers using a common denominator.
Step 2 Divide the numerators, and divide the denominators.

Solve. Show your work.

3. $5 \div \frac{2}{3} = \underline{\text{______}}$

4. $\frac{4}{7} \div \frac{3}{5} = \underline{\text{______}}$

5. $4\frac{1}{8} \div \frac{3}{4} = \underline{\text{______}}$

6. $6\frac{2}{3} \div \frac{7}{9} = \underline{\text{______}}$

Practice

7. $4\frac{1}{4} = 3\underline{\text{______}}$

8. $\underline{\text{______}} = 3\frac{7}{5}$

9. $1\frac{3}{5} + 2\frac{1}{5} = \underline{\text{______}}$

10. $3\frac{3}{8} - 1\frac{5}{8} = \underline{\text{______}}$

11. $7\frac{4}{9} - 5\frac{8}{9} = \underline{\text{______}}$

12. $3\frac{2}{7} + 1\frac{4}{5} = \underline{\text{______}}$

13. $5\frac{2}{3} + 2\frac{3}{4} = \underline{\text{______}}$

14. $4 - 1\frac{3}{4} = \underline{\text{______}}$

15. $3 \times 3\frac{3}{4} = \underline{\text{______}}$

16. $4\frac{2}{3} \times 6\frac{7}{7} = \underline{\text{______}}$

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Solving Mixed-Number Addition Problems

Add. Write each sum as a mixed number in simplest form. Show your work.

1. \[5 \frac{1}{5} + 2 \frac{4}{5} = \]  
2. \[3 \frac{2}{5} + 5 \frac{3}{10} = \]

3. \[4 \frac{3}{4} + 2 \frac{1}{12} = \]  
4. \[4 \frac{2}{3} + 2 \frac{3}{4} = \]

5. Josiah was painting his garage. Before lunch, he painted \(1 \frac{2}{3}\) walls. After lunch, he painted another \(1 \frac{2}{3}\) walls. How many walls did he paint during the day?

6. Julie’s mom made muffins for Julie and her friends to share. Julie ate \(1 \frac{3}{4}\) muffins. Her friends ate \(3 \frac{1}{2}\) muffins. How many muffins did Julie and her friends eat altogether?

Without adding the mixed numbers, insert <, >, or =. Explain how you got your answer.

7. \[1 \frac{3}{8} + 6 \frac{2}{3} \] \(\leq\) \(8\)

8. \[5 \leq 2 \frac{1}{5} + 2 \frac{7}{8}\]
LESSON 8.12
Number Stories: Division with Fractions

1. Five pies will each be sliced into fourths. Ira would like to find out how many slices there will be in all.
   a. Show how the pies will be cut.
   b. The drawings show that $5 \div \frac{1}{4} = \underline{\hspace{2cm}}$, so there will be \underline{\hspace{2cm}} slices in all.

2. Jake has a 3-inch strip of metal. He would like to find out how many $\frac{1}{2}$-inch strips he can cut.
   Jake can cut \underline{\hspace{2cm}} strips. So, $3 \div \frac{1}{2} = \underline{\hspace{2cm}}$.

3. Two students equally share $\frac{1}{4}$ of a granola bar. They would like to know how much of the bar each will get.
   a. Show how the piece of granola bar will be cut.
   b. The drawing shows that $\frac{1}{4} \div 2 = \underline{\hspace{2cm}}$, so each student will get \underline{\hspace{2cm}} of a granola bar.

4. a. Drawing A can be used to find $\frac{1}{3} \div 5$.
   Drawing B can be used to find $\frac{1}{3}$ of $\frac{1}{5}$, or $\frac{1}{3} \times \frac{1}{5}$. Use the drawings to show that $\frac{1}{3} \div 5 = \frac{1}{3} \times \frac{1}{5}$.
   b. Complete.
      \[ \frac{1}{3} \times \frac{1}{5} = \underline{\hspace{2cm}} \]
      \[ \frac{1}{3} \div 5 = \underline{\hspace{2cm}} \]
      \[ \frac{1}{3} \div 5 = \frac{1}{3} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}} \]
Rulers

0 1 2 3 4 5 6
inches

0 1 2 3 4 5 6
inches

0 1 2 3 4 5 6
inches