Clock Fractions and Common Denominators

Objective: To provide additional references for fraction concepts.

Technology Resources: www.everydaymathonline.com

Key Concepts and Skills
- Find common denominators. [Number and Numeration Goal 5]
- Use clock models and pencil-and-paper algorithms to add and subtract fractions. [Operations and Computation Goal 4]
- Use benchmarks to estimate sums and differences. [Operations and Computation Goal 6]

Key Activities
Students use a clock face to find equivalent fractions and to model addition and subtraction of fractions. They use the multiplication rule, a multiplication table, and reference lists to find common denominators.

Ongoing Assessment:
Recognizing Student Achievement
Use page 194. [Operations and Computation Goal 4]

Key Vocabulary
common denominator  Unlike denominators

Materials
Math Journal 1, pp. 194–197
Student Reference Book, p. 401
Study Link 6-8
Math Masters, p. 177
Multiplication Table Poster  Demonstration clock

Advance Preparation
For Part 1, display the Multiplication Table Poster on the board or elsewhere for student use. When you use the Everyday Mathematics posters with English language learners, display either the English version only or both English and Spanish versions simultaneously; do not display the Spanish version only.


1 Teaching the Lesson
2 Ongoing Learning & Practice
3 Differentiation Options

Playing Fraction Capture
Math Journal 1, p. 198
Math Masters, p. 460
per partnership: 2 six-sided dice
Students practice comparing fractions and finding equivalent fractions.

Math Boxes 6-9
Math Journal 1, p. 199
Students practice and maintain skills through Math Box problems.

Study Link 6-9
Math Masters, p. 178
Students practice and maintain skills through Study Link activities.

ENRICHMENT
Modeling Fractions with a Military Clock
Math Masters, p. 179
Students apply the clock model to a different unit by using military time.

EXTRA PRACTICE
Writing Elapsed Time Number Stories
Math Masters, p. 180
Students write a number story using fractions to represent elapsed time.
Getting Started

Mental Math and Reflexes

Fraction addition and subtraction: Write the problems on the board, and have students estimate using benchmarks and then solve. Students use the estimate to assess the reasonableness of the answers.

\[
\begin{align*}
\frac{5}{8} + \frac{3}{4} + 2 \frac{1}{2} &= ? \frac{3}{8} \\
\frac{1}{4} + \frac{1}{2} + 1 \frac{1}{4} &= ? 6 \\
\frac{6}{3} - 4 + 2 \frac{1}{4} &= ? 4 \frac{1}{3} \\
\frac{5}{3} + \frac{1}{5} + 3 \frac{3}{4} &= ? 6 \frac{7}{16} \\
\frac{3}{8} + 2 - 1 \frac{3}{12} + 1 \frac{1}{2} &= ? 6 \frac{1}{8}
\end{align*}
\]

Math Message

Complete Part 1 on journal page 194.

Study Link 6-8 Follow-Up

Have partners compare answers and resolve differences.

NOTE Working with elapsed time can provide students with more practice using a clock face to add and subtract fractions. Remind students that elapsed time is the time that passes between a given starting and ending time. Give them various problems that involve finding elapsed time and then adding or subtracting the amounts. Ask students to give the elapsed time in minutes and then in fractions of an hour. They can then use the clock face to add or subtract.

1 Teaching the Lesson

Math Message Follow-Up

(Math Journal 1, p. 194)

Review the answers to Part 1 with the class. You might want to pose a few additional easy problems that have mixed numbers or fractions greater than 1. Suggestions:

- How many minutes are in 2 1/2 hours? 150 min
- In 5/2 hours? 150 min
- In 5/4 hours? 75 min
- 3/2 hours is equivalent to how many minutes? 90 min

Using a Clock to Add and Subtract Fractions

(Math Journal 1, p. 194)

Make sure students understand that they may use the clock model to help them answer the problems on the journal page. At times, students might want to “think minutes,” as in the example for Part 3. At other times, students might want to look at the clock face divided into twelfths. Using a demonstration clock, work several of the problems in Part 2 with the class before students work in partnerships. Assign the remainder of the journal page.

Ongoing Assessment: Recognizing Student Achievement

Use journal page 194, Problems 16–24 to assess students’ ability to use a visual model to add and subtract fractions with unlike denominators. Students are making adequate progress if they correctly solve Problems 16–24.

[Operations and Computation Goal 4]
Discussing Strategies for Adding and Subtracting Fractions

(Math Journal 1, p. 194)

Discuss students’ solutions to Part 3. Expect that some students converted most fractions to minutes, did the operation, and then converted the answer in minutes back to a fraction. Others may have converted all fractions to twelfths and found the answer without any reference to the clock or time.

Adjusting the Activity

Pose fraction problems with denominators of 30 and 60.

Suggestions follow:
- \( \frac{3}{4} + \frac{5}{60} \) or \( \frac{5}{6} \)
- \( \frac{18}{30} + \frac{28}{30} \) or \( \frac{14}{15} \)
- \( \frac{5}{12} + \frac{25}{60} \) or \( \frac{5}{6} \)

NOTE A clock face is a convenient model for fraction operations involving halves, thirds, fourths, fifths, sixths, twelfths, and even thirtieths and sixtieths. The link between fractions and their equivalents in minutes allows students to add and subtract fractions with unlike denominators without rewriting the fractions with a common denominator.

Using a Multiplication Table to Explore Equivalent Fractions

(Math Journal 1, p. 195; Math Masters, p. 177)

Any two rows of a multiplication table can be used to form equivalent fractions. Display the Multiplication Table Poster on the board. Ask partners to cut the strips from Math Masters, page 177 and place them in the middle of their workspace.

- Each student takes one strip. Ask students to make true statements about the numbers on their strip. The numbers are multiples of the first number on the strip; the numbers have a common factor. Tell them that a strip can be named by its smallest number, for example, the “4 strip.”
- One partner is “numerator” and the other is “denominator.” Partners then match their strips, laying the numerator strip above the denominator strip. Tell students that the columns form fractions.
- For Problem 1, on journal page 195, ask students to write down the strip names and then list all of the fractions formed by the matches.
- Partners then take two different strips and repeat this process for Problems 2 and 3. Each strip should be used only once.

Circulate and assist.

Ask students what they notice about their lists of fractions. The numerators and denominators are multiples of the numbers in the first column; the fractions are equivalent.
Study the examples. Then work the problems below in the same way.

**Example 1:**

- **Unlike Denominators:** \(\frac{2}{3} - \frac{1}{2}\)
- **Common Denominators:** \(\frac{4}{6} - \frac{3}{6}\)
- **Common Denominator:** \(\frac{1}{6}\)

**Example 2:**

- **Unlike Denominators:** \(\frac{3}{4} - \frac{1}{2}\)
- **Common Denominators:** \(\frac{6}{8} - \frac{4}{8}\)
- **Common Denominator:** \(\frac{2}{8}\)

Ask volunteers to match two of their remaining strips and write the fraction from the first column on the board. Use these fractions to demonstrate the multiplication rule.

**Example:**

\[
\frac{4}{9} = \frac{4 \times 2}{9 \times 2} = \frac{8}{18}
\]

Ask students to use the appropriate strips to give another equivalent fraction for \(\frac{4}{9}\). Then ask a volunteer to write the number model for this change using the multiplication rule.

**Sample response:** \(\frac{32}{72} = \frac{4 \times 8}{9 \times 8}\)

Refer students to the Multiplication Table Poster. Explain that for any two rows, the equivalent fractions are the result of multiplying the fraction in the first column by another name for 1, such as \(\frac{2}{2}\) or \(\frac{8}{8}\), depending on the column. So the second column is the result of multiplying by \(\frac{2}{2}\), the third column is the result of multiplying by \(\frac{3}{3}\), and so on.

### Using a Common Denominator

(Math Journal 1, pp. 196 and 197; Student Reference Book, p. 401)

**Algebraic Thinking** Introduce the next activity by discussing the following points:

- It is easy to add or subtract fractions if they have the same denominator, usually called a **common denominator**. To support English language learners, discuss the meaning of **common** in this mathematical context.

- One way to add or subtract fractions with different denominators, usually called **unlike denominators**, is to rewrite the fractions with a common denominator.

- One way to find common denominators is to use the multiplication rule (or the division rule) for finding equivalent fractions. Ask volunteers to express the rules with variables.

\[
\frac{a}{b} = \frac{a \times n}{b \times n}, \quad \frac{a}{b} = \frac{a \div n}{b \div n}
\]

Have students look at Example 1 and Example 2 at the top of Math Journal 1, page 196. Ask: *How could you use benchmarks to estimate the solution to each problem?* Sample answer: For Problem 1, I know that \(\frac{3}{8}\) is greater than \(\frac{1}{2}\) and \(\frac{1}{6}\) is less than \(\frac{1}{2}\), so the answer will be close to 1. For Problem 2, I know that \(\frac{3}{8}\) and \(\frac{3}{4}\) are both close to 1, so my answer will be close to zero.

**NOTE** Some students may realize that \(\frac{3}{8}\) is less than \(\frac{1}{2}\) and conclude that the answer will be less than 1.

Then work through the examples to illustrate the use of the multiplication rule to find common denominators. Pose one or two similar problems as needed.
In addition to using the multiplication rule to find equivalent fractions, students can also refer to the Table of Equivalent Fractions, Decimals, and Percents on page 401 of the Student Reference Book.

Assign journal pages 196 and 197. Remind students of the importance of using benchmarks to estimate the solution and then assess the reasonableness of their answers. Students may choose to solve Problems 7 and 8 by finding a common denominator.

**NOTE** In Problems 1, 2, 5, 6, 7, and 8, on journal pages 196 and 197, the common denominator is the same as one of the original denominators. In Problems 3 and 4, the common denominator is different from both of the original denominators.

## Ongoing Learning & Practice

### Playing Fraction Capture

*Math Journal 1, p. 198; Math Masters, p. 460*

Players roll dice, form fractions, and claim corresponding sections of squares. The rules are on Math Journal 1, page 198, and the gameboard is on Math Masters, page 460. Remind students of the importance of using the benchmark fraction of $\frac{1}{2}$ when playing this game.

### Math Boxes 6-9

*Math Journal 1, p. 199*

**Mixed Practice** Math Boxes in this lesson are paired with Math Boxes in Lesson 6-6. The skill in Problem 6 previews Unit 7 content.

**Writing/Reasoning** Have students write a response to the following: Explain your answer to the question in Problem 3 and how you chose the values for the data set.

Because the average cannot be greater than the maximum in the data set, 53 inches cannot be Esther's average since 50 is the maximum number. I chose 5 numbers for the data set that could be added together and divided by 5 so that the average would equal 53.

### Study Link 6-9

*Math Masters, p. 178*

**Home Connection** Students solve problems similar to those on journal pages 196 and 197. This page reinforces the idea that a common denominator can be determined by finding fractions equivalent to the given fractions.
Differentiation Options

ENRICHMENT

Modeling Fractions with a Military Clock

(Math Masters, p. 179)

To apply students’ understanding of the fractional units on a 12-hour clock face, have students use a 24-hour military clock face model to add, subtract, and find equivalent fractions. When they have finished the page, have students describe similarities and differences between using the 12-hour clock and the 24-hour clock.

EXTRA PRACTICE

Writing Elapsed Time Number Stories

(Math Masters, p. 180)

Students write a number story using fractions to represent elapsed time. Ask students to exchange and solve each other’s problems and then share their solution strategies.