Comparing Millions, Billions, and Trillions

Objectives To provide experience with comparing the relative sizes of 1 million, 1 billion, and 1 trillion and using a sample to make an estimate.

1 Teaching the Lesson

Key Concepts and Skills
• Read and write large numbers. [Number and Numeration Goal 1]
• Compare order of magnitude for large numbers. [Number and Numeration Goal 6]
• Make reasonable estimates for whole number multiplication problems. [Operations and Computation Goal 6]

Key Activities
Students review time conversion factors. They count the number of times they can tap their desks in 10 seconds and estimate how long it would take to tap 1 million times. Students then estimate how long it would take to tap 1 billion and 1 trillion times.

Ongoing Assessment: Informing Instruction See page 134.

Key Vocabulary
sample

Materials
Math Journal 1, p. 57
Study Link 2-10
Class Data Pad ◆ blank paper or construction paper ◆ markers or crayons ◆ watch or timer with second hand ◆ calculator

2 Ongoing Learning & Practice

Playing High-Number Toss
Student Reference Book, p. 320
Math Masters, p. 487 per partnership: 1 die ◆ 1 sheet of paper
Students practice concepts of place value and standard notation by writing and comparing large numbers.

Ongoing Assessment: Recognizing Student Achievement
Use Math Masters, page 487. [Numbers and Numeration Goals 1 and 6]

Math Boxes 2-10
Math Journal 1, p. 58
Students practice and maintain skills through Math Box problems.

Study Link 2-10
Math Masters, p. 61
Students practice and maintain skills through Study Link activities.

3 Differentiation Options

READINESS
Playing Number Top-It (7-Digit Numbers)
Student Reference Book, p. 326
Math Masters, pp. 491 and 492 per partnership: 4 each of number cards 0–9 (from the Everything Math Deck, if available)
Students apply place-value concepts to form, read, and compare large numbers.

EXTRA PRACTICE
Comparing Powers of 10 Using Place Value
Math Masters, p. 66B
Students apply place value for powers of 10.

ENRICHMENT
Applying Estimation Strategies
Math Masters, p. 62
Students make time estimates and identify the number models used for their estimation strategies.

Advance Preparation
For Part 1, use the Class Data Pad to record and display the Mental Math and Reflexes problems and responses. For the optional Readiness activity in Part 3, make one game mat for each partnership by copying, cutting, and taping together Math Masters, pages 491 and 492.

Getting Started

**Mental Math and Reflexes**

Have students practice conversions between units of time. Use the Class Data Pad to record the correct responses. Keep this display up for students to refer to during the lesson.

<table>
<thead>
<tr>
<th>How many seconds are in...</th>
<th>How many minutes are in...</th>
<th>How many hours are in...</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 minute? 60</td>
<td>1 hour? 60</td>
<td>1 day? 24</td>
</tr>
<tr>
<td>3 minutes? 180</td>
<td>5 hours? 300</td>
<td>2 days? 48</td>
</tr>
<tr>
<td>100 minutes? 6,000</td>
<td>50 hours? 3,000</td>
<td>10 years? about 3,650</td>
</tr>
</tbody>
</table>

**Math Message**

Explain the strategy you would use to find the number of minutes in one year.

**Study Link 2-9**

Follow-Up

Have partners share answers and resolve any differences.

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**1 Teaching the Lesson**

**Math Message Follow-Up**

Have students in small groups discuss their individual strategies. The group then decides which steps they would use to find the number of minutes in a year. Each group should make a poster, using construction paper and markers or crayons, to list their steps and place the poster on display. Allow time for students to read the displayed group posters. Ask volunteers to identify the similarities and differences in the strategies. Guide the class discussion to focus on summarizing the strategies. Sample answers: Convert a year into a unit of time that can be converted to minutes; convert a year into days; convert these days into hours and the hours into minutes.

**NOTE** Include a walkabout in the follow-up to this Math Message: Display the group posters in separate areas of your classroom, and allow students time to browse until they have read all the posters.

**Solving a Tapping Problem with Sampling Strategies**

(Math Journal 1, p. 57)

Ask students to refer to the Useful Information chart on journal page 57. Pose several questions from the first row of information to highlight these large number relationships for students. What is 1,000 times 1 million? 1 billion One billion has what relationship to 1 trillion? One trillion is 1,000 times 1 billion. Ask a volunteer to read the question labeled Make a guess. Ask students to guess how long it would take them to tap their desks 1 million times without any interruptions. Have students...
Player 1 rolls the die and writes the number on any of his or her blank lines. Player 2 rolls the die and writes the number on one of his or her blank lines. The number on the last blank tells the number of zeros that come after the first 3 digits. Each player draws 4 blank lines on a sheet of paper to record the numbers that they build a number. The numbers on the first 3 blanks are the first 3 digits of the number the player builds. The numbers on the last 3 blanks are the first 3 digits of the number the player builds. Each player needs 4 or her number. (See the place-value chart below.) The place value chart tells you how to round the number that comes after the first 3 digits.

Object of the game: To make the largest number possible.

Skill: Place value, exponential notation

Directions:
1. Each player draws 4 blank lines on a sheet of paper to record the numbers that come after the first 3 digits.
2. Player 1 rolls the die and writes the number on any of his or her blank lines. It does not have to be the first blank—it can be any of them. Keep in mind that the larger number wins!
3. Player 2 rolls the die and writes the number on one of his or her blank lines.
4. Players take turns rolling the die and writing the number 3 more times each.
5. Each player then uses the 6 numbers on his or her blank lines to build a number.

Sample

| Number | Ones | Tens | Hundreds | Thousands | Ten Thousands
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Player 1: 123,456</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Player 2: 56,000,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Math Masters, p. 487

Students might suggest strategies such as the following:

- Count how many times you can tap your desk in a set amount of time, such as 10 seconds.
- Time how long it takes you to tap a certain number of times, such as 100 times.
- Pick a reasonable number of taps for a set amount of time, and make an estimate based on that rate, such as 3 taps per second.

When the class found and used the median step length, they were using a sample. Ask whether students know of any other situations where samples are used.

Ongoing Assessment: Informing Instruction

Watch for students who are having difficulty developing a strategy. Explain that this is another situation for which obtaining the exact answer is impossible, such as the Estimation Challenge from Lesson 2-1. Recall the strategies students used in that lesson and have them use similar approaches here.

Math Masters, p. 478

- Count how many times you can tap your desk in a set amount of time, such as 10 seconds.
- Time how long it takes you to tap a certain number of times, such as 100 times.
- Pick a reasonable number of taps for a set amount of time, and make an estimate based on that rate, such as 3 taps per second.

When the class found and used the median step length, they were using a sample. Ask whether students know of any other situations where samples are used.

NOTE A sample of anything is a small piece or part that is intended to give information about the whole thing. Consumers use product samples to decide whether products suit their needs. Pollsters use population samples to estimate information for the whole population. The finger-tapping samples here are time samples: The count of taps in a 10-second period is a sample used to determine a tapping rate, and the rate is then used to estimate how long it would take to make large numbers of taps.

Each student will take a 10-second sample count of their own finger tapping. Practice taking sample counts by timing students as they tap and count for 10 seconds.
Using Sampling to Make an Estimate

(Math Journal 1, p. 57)

Have partners complete the journal page. They begin by finding their individual sample counts. Partners take turns. While one partner taps and counts the taps for 10 seconds, the second partner keeps time for 10 seconds, signaling when to start and stop.

Partners then use their 10-second sample counts to estimate the number of taps they could make in 1 minute; in 1 hour; and in 1 day. Encourage students to use their calculators, as needed.

Next students use their estimates to calculate the approximate number of days it would take to tap 1 million times. Encourage students to devise their own solution strategies. One possible approach is to divide 1 million by the number of taps per day.

Making Time Estimates for 1 Billion and 1 Trillion Taps

(Math Journal 1, p. 57)

In Problem 3 on journal page 57, students estimate the time it would take to tap 1 billion and 1 trillion times. Remind students that they can use the relationships between 1 million, 1 billion, and 1 trillion found in the Useful Information chart to help them estimate. They will also need to decide whether to report their estimates for 1 billion and 1 trillion taps as days or years.

NOTE: Expect that the tapping rate for most students will be about 40 times in 10 seconds. At this rate they will tap about 250 times in 1 minute (6 * 40, rounded up); 15,000 times in 1 hour (60 * 250); 350,000 times in 1 day (24 * 15,000; rounded down), and about 3 days, without interruptions, to tap 1 million times.

Sharing and Discussing the Results

(Math Journal 1, p. 57)

When most students have completed the problems, have partners form small groups to discuss their strategies. Then have the groups report on the similarities and differences of the strategies used as well as any notable experiences they encountered. Use the following questions as a guide:

- How does your estimate of the time for 1 million taps compare with your initial guess?
- Did you use your estimate for the number of taps in 1 day to estimate how long it would take to tap 1 million times?
- Did you use the time for 1 million taps to estimate the time for 1 billion and 1 trillion taps?
To conclude Part 1 ask students: **If one person could tap 24 hours per day without stopping, would it be possible to tap 1 trillion times?** No Aim follow-up questions at getting students to support their responses. They would still need many more years than are in a normal lifetime. An exit question might be: **Do you feel that your informed estimate was more reasonable than your guess?**

### Ongoing Learning & Practice

#### Playing High-Number Toss

(Student Reference Book, p. 320; Math Masters, p. 487)

**High-Number Toss** provides students with the opportunity to apply their knowledge of place value and standard notation to create, write, read, and compare large numbers. Provide students with a reminder box on the board noting that \(<\) means **less than** and \(>\) means **greater than**.

### Ongoing Assessment: Recognizing Student Achievement

Use the Record Sheet for **High-Number Toss** (Math Masters, page 487) to assess students’ knowledge of place value and comparing numbers. Students are making adequate progress if they correctly insert the relational symbols between the two numbers in five rounds of the game.

[Numbers and Numeration Goals 1 and 6]

### Math Boxes 2·10

(Math Journal, p. 58)

**Mixed Practice** Math Boxes in this lesson are paired with Math Boxes in Lesson 2-8. The skill in Problem 4 previews Unit 3 content.

### Study Link 2·10

(Math Masters, p. 61)

**Home Connection** Students use their knowledge of place value and number relationships to solve number puzzles.
Differentiation Options

**REACHNESS**

- **Playing Number Top-It (7-Digit Numbers)**
  (Student Reference Book, p.326; Math Masters, pp. 491 and 492)

To review place-value concepts, have students play Number Top-It (7-Digit Numbers).

**EXTRA PRACTICE**

- **Comparing Powers of 10 Using Place Value**
  (Math Masters, p. 66B)

To provide additional practice with place value and understanding the relationships between powers of 10, have students complete Math Masters, page 66B.

**ENRICHMENT**

- **Applying Estimation Strategies**
  (Math Masters, p. 62)

To apply students’ ability to use estimation strategies, have them solve problems that involve converting situational information into open number sentences. Direct students to focus on making informed estimates.
1 meter 10 decimeters 100 centimeters 1,000 millimeters
1 centimeter 0.01 meter 0.1 decimeter 10 millimeters

Use the information in the conversion table to respond to each statement below. Complete each statement with one of the following phrases:
10 times, 100 times, $\frac{1}{10}$ of, $\frac{1}{100}$ of

1. 1 meter is ________________ the size of a decimeter.
2. 1 centimeter is ________________ the size of a meter.
3. 1 centimeter is ________________ the size of a millimeter.
4. 1 decimeter is ________________ the size of a meter.
5. 1 millimeter is ________________ the size of a decimeter.

Write two of your own statements using the information in the table.
6. ____________________________________________
7. ____________________________________________

Complete the table below by making the appropriate conversions.

<table>
<thead>
<tr>
<th>millimeters</th>
<th>centimeters</th>
<th>decimeters</th>
<th>meters</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.</td>
<td>9,743</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>175</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

11. In Problem 10, explain what happens to the value of the digit 5 when you go from millimeters to centimeters, and then from decimeters to meters.