12-1 Factor Trees

Objective To provide experiences with finding the greatest common factor and the least common multiple of two numbers.

Key Concepts and Skills
• Identify the prime factorization for a number. [Number and Numeration Goal 3]
• Use greatest common factors and least common multiples to rename fractions. [Number and Numeration Goal 5]
• Use multiplication facts to find factor strings. [Operations and Computation Goal 2]
• Find greatest common factors and least common multiples using factor strings. [Number and Numeration Goal 3]

Key Activities
Students use factor trees to find all the prime factors of a number and write the prime factorization. They use prime factorizations to simplify fractions.

Ongoing Assessment: Recognizing Student Achievement
Use journal page 394. [Number and Numeration Goal 3]

Key Vocabulary
prime factorization • factor tree • common factor • greatest common factor • least common multiple

Materials
Math Journal 2, pp. 393–396
Student Reference Book Glossary
Class Data Pad

Playing Factor Captor
Student Reference Book, p. 306
Math Masters, p. 455
per partnership: 70 counters
Students practice finding factors of larger numbers and recognizing prime factors.

Math Boxes 12-1
Math Journal 2, p. 397
Students practice and maintain skills through Math Box problems.

Study Link 12-1
Math Masters, p. 348
Students practice and maintain skills through Study Link activities.

READINESS
Making Factor Rainbows
Students factor numbers by making factor rainbows.

ENRICHMENT
Using a Division Method for Prime Factorizations
Math Masters, p. 349
Students use division to find prime factorizations.

EXTRA PRACTICE
Using Factor Trees to Find Common Denominators
Math Masters, p. 350
Students identify common denominators by using factor trees.

ELL SUPPORT
Making a Factor Tree Poster
per group: chart paper, markers
Students make a display to reference concepts related to factor trees.

Advance Preparation
Teacher’s Reference Manual, Grades 4–6 pp. 79–82
Getting Started

Math Message Follow-Up
(Math Journal 2, p. 393; Student Reference Book Glossary)

Have students read the definitions for factor, factor pair, composite number, prime number, and factor string in the Student Reference Book Glossary. Read and discuss how the terms relate. Discussion should include the following points:

- A factor of a number can be any type of number, but the other terms only refer to whole numbers.
- Numbers can be defined by their factors. For example, a prime number has only two whole-number factors, 1 and itself. A composite number has more than two whole-number factors. A square number has an odd number of factors.
- Each factor in the longest factor string of a number is prime. The longest factor string is called its prime factorization.

Ask: What device can be used to find all the factor pairs for a number?

**Factor rainbow** Have students make factor rainbows for 48 and share their solutions using the Class Data Pad. A factor tree is a device that can be used to find the prime factorization of a number. Draw one for 48 on the Class Data Pad. Ask: What is the prime factorization for 48? 2 * 2 * 2 * 2 * 3 Have students refer to the factor tree examples for 45 on Math Journal 2, page 393. Note that each factor tree begins with two different factors for 45 but the prime factorizations are the same. Ask students to make factor trees for 36. Ask: What do you notice about the longest factor string you recorded for Problem 2 and the result of your factor tree for 36? They are the same: 2 * 2 * 3 * 3.

Finding Greatest Common Factors
(Math Journal 2, pp. 393–395)

Write the numbers 18 and 30 on the board. Ask students to name all the factors of each number while you list them.
Find the greatest common factor of 24 and 60.

Make factor trees and write the prime factorization of each number.

What is the greatest common factor of 75 and 90?

What is the greatest common factor of 20 and 25?

Use the factor trees in Problem 4 to help you write each fraction below in simplest form. Divide the numerator and denominator by their greatest common factor.

Which prime factor(s) do 10 and 75 have in common?

What is the greatest common factor of 10 and 90?

Use the space below to draw factor trees. What common factors do these two numbers have in common, and multiply them.

Ask students to make factor trees for 18 and 30, and have volunteers draw these two factor trees on the board.

Write the prime factorizations for 18 and 30 as shown below (one above the other) on the board or a transparency. Ask: What prime factors do these two numbers have in common? 2 and 3 Circle the pairs of common factors in the prime factorizations for 18 and 30. Explain that since 2 * 3 = 6, the greatest common factor is 6. It is the greatest number by which both 18 and 30 are divisible.

Have students make factor trees for 24 and 50, and have volunteers add these two trees to the display on the board. Ask: What is the greatest common factor of 24 and 50? 2 How do the factor trees confirm that 2 is the greatest common factor of 24 and 50? They show that 2 is the only common prime factor.

Have students refer back to the factor trees for 24 and 36. Ask: What is the greatest common factor of 24 and 36? 2 * 2 * 3, or 12 Write the prime factorizations for 24 and 36, circling the pairs of common factors. Point out that 2, 2, and 3 are the common prime factors. Therefore, the greatest common factor is 2 * 2 * 3, or 12.

Tell students that the greatest common factor can be used to simplify fractions. Ask students how they might use the greatest common factor and the division rule to simplify the fraction. The factor trees for 24 and 36 show their greatest common factor to be 12. Divide the numerator and denominator by the greatest common factor, \( \frac{24}{12} \div \frac{36}{12} = \frac{2}{3} \). The simplified fraction is \( \frac{2}{3} \). Have students complete journal pages 393–395.

Factors of 18: 1, 2, 3, 6, 9, and 18

Factors of 30: 1, 2, 3, 5, 6, 10, 15, and 30

Circle 1, 2, 3, and 6 on both lists. Explain that because 1, 2, 3, and 6 are factors of both 18 and 30, they are called common factors.

The largest of these common factors is called the greatest common factor. The greatest common factor of 18 and 30 is 6.

There are two ways to find the greatest common factor of two numbers:

- List all the factors of each number and identify the largest common factor.
- Use factor trees to write the prime factorization of each number. Identify the prime factors that these two numbers have in common, and multiply them.
Ongoing Learning & Practice

Finding Least Common Multiples
(Math Journal 2, p. 396; Student Reference Book Glossary)

Have students locate the term least common multiple in the Student Reference Book Glossary and read the entry. Ask: What is a common multiple? A number that is a multiple of two or more given numbers The quick common denominator is an example of a common multiple. The smallest number that is a multiple of two or more numbers is the least common multiple.

Have students list the first seven multiples of 6 and 9. Ask: What numbers are in both lists? The common multiples are 18 and 36. What is the least common multiple of 6 and 9? 18

Tell students that factor trees also can be used to find the least common multiple of two numbers. This procedure is similar to finding the greatest common factor.

Step 1: Refer to the prime factorizations for 18 and 30 from the previous activity, circling the pairs of common factors.

Step 2: Draw a line through one factor in each of the pairs of common factors.

18 = 2 × 3 × 3
30 = 2 × 3 × 5

Least common multiple: 2 × 3 × 3 × 5 = 90

Step 3: Write the remaining factors in a multiplication expression, 2 × 3 × 3 × 5 = 90. The least common multiple of 18 and 30 is 90.

Have partners complete journal page 396. Circulate and assist.

Ongoing Learning & Practice

Playing Factor Captor
(Student Reference Book, p. 306; Math Masters, p. 455)

Students practice finding factors of larger numbers and recognizing prime factors by playing Factor Captor on the 1–110 Grid.
Math Boxes 12-1
(Math Journal 2, p. 397)

Mixed Practice Math Boxes in this lesson are paired with Math Boxes in Lesson 12-3.

Writing/Reasoning Have students write a response to the following: Explain your solution strategy for Math Boxes, Problem 2. Since $120 represents 30%, I divided 120 by 3 to find 10% of the list price. Then I multiplied that result by 10 to find the total list price; $120 ÷ 3 * 10 = $280.00.

Study Link 12-1
(Math Masters, p. 348)

Home Connection Students work with factor trees. They use these to write prime factorizations and to write fractions in simplest form.

3 Differentiation Options

Making Factor Rainbows
To provide experience finding all the factors of a number, have students make factor rainbows. For example, to make a factor rainbow for the number 48, list all the factors of 48 in ascending order. Then connect factor pairs. Every factor should be paired with another factor. If there is an odd number of factors, the middle factor is paired with itself. The product of each pair of factors should be 48.

Have students make factor rainbows for given numbers. Suggestions: 25, 32, 40, 49, 80, 100

Using a Division Method for Prime Factorizations
(Math Masters, p. 349)

To apply students’ understanding of factors, have them use a division method to find prime factorizations. When students have finished, discuss any questions or curiosities they encountered.
Using Factor Trees to Find Common Denominators

(Math Masters, p. 350)

Students practice making factor trees to find prime factorizations. They use factor trees to find common denominators, identify the least common multiple of the denominators, and use the least common multiple as a common denominator.

Making a Factor Tree Poster

To provide language support for factors, guide students to make a poster showing important vocabulary words related to factor trees. Display the poster during lessons in this unit.

Factors

Factors: 1, 2, 3, 4, 6, 12

Factor Tree:

Factors: 1, 2, 4, 5, 10, 20

Greater Common Factor for 12 and 20 is 4

Least Common Multiple for 12 and 20 is 60

(3 * 2 * 2 * 2 * 5 = 60)