Lesson 10-2

Pan-Balance Problems with Two Balances

Objective To develop a pan-balance approach for solving sets of two equations with two unknowns.

Key Concepts and Skills

- Use addition and subtraction to solve pan-balance problems. [Operations and Computation Goal 1]
- Use multiplication and division to solve pan-balance problems. [Operations and Computation Goal 3]
- Use a pan-balance model to solve linear equations with two unknowns. [Patterns, Functions, and Algebra Goal 2]

Key Activities

Students solve pan-balance problems involving two balances.

Ongoing Assessment: Recognizing Student Achievement

Use the Math Message. [Patterns, Functions, and Algebra Goal 2]

Materials

Math Journal 2, pp. 336–338
Study Link 10-1
slate for demonstration purposes:
100 standard 1" paper clips, 3 identical ballpoint pens, 3 identical compasses (pencils removed), 2 identical plastic 6" half-circle protractors, 1 plastic 12" ruler, 1 or 2 pan balances

Advance Preparation

For Part 1, use two pan balances for demonstration purposes. Alternately, the lesson includes instructions for a single pan balance.


Making and Interpreting Line Plots

Math Journal 2, p. 339
Students construct and interpret a line plot.

Math Boxes 10-2

Math Journal 2, p. 340
Students practice and maintain skills through Math Box problems.

Study Link 10-2

Math Masters, p. 297
Students practice and maintain skills through Study Link activities.

Solving Logic Puzzles

Class Data Pad (optional)
Students solve logic puzzles related to pan-balance problems.

Measuring Time

Math Masters, p. 298
Students explore a sandglass model to solve a problem about time intervals.
More Pan-Balance Problems

Math Message ★

LESSON 10 /a51.001

Date  Time

Math Message

Solve these pan-balance problems. In each figure, the two pans are in perfect balance.

1. One block weighs as much as 2 balls.
2. One ball weighs as much as 5 marbles.
3. One block weighs as much as 13 marbles.
4. One block weighs as much as 30 marbles.
5. One block weighs as much as 9 marbles.

Solve these problems using both pan balances. In each problem, the pans are in perfect balance. The weights of objects, such as blocks, balls, marbles, and coins, are consistent within each problem.

1. One block weighs as much as 2 balls. One ball weighs as much as 5 marbles. Therefore, one block weighs as much as 10 marbles.
2. Because one ball weighs as much as 5 marbles, 4 balls weigh as much as 20 marbles. Therefore, 2 blocks weigh as much as 20 marbles, and 1 block weighs as much as 10 marbles.

Math Message Follow-Up

(Math Journal 2, p. 336)

Draw the Math Message pan-balance problems on the board or a transparency. Have volunteers share their answers and solution strategies. Under the pan balances, write: One block weighs as much as how many marbles? Ask: Can you solve this problem using only one of the pan balances? No Discuss how the information from both pan balances needs to be combined to solve the problem.

One block weighs as much as 2 balls. One ball weighs as much as 5 marbles. Therefore, one block weighs as much as 10 marbles.

Because one ball weighs as much as 5 marbles, 4 balls weigh as much as 20 marbles. Therefore, 2 blocks weigh as much as 20 marbles, and 1 block weighs as much as 10 marbles.

Demonstrating How to Solve More Complex Pan-Balance Problems

Explain that the class will look at two other examples where the information from two pan balances needs to be combined to solve the problems.
Example 1:
Show the protractor, ruler, and paper clips. Tell students their goal is to determine the weight of the ruler and the protractor in terms of paper clips. (See margin.)

Set up the first pan balance. Place a protractor in one pan. Then add paper clips to the other pan, one at a time, counting as you go, until the pans balance. If you have only one balance available, sketch the results of the first pan-balance situation on the board before setting up the second balance.

Set up the second pan balance. Place a protractor and ruler in one pan. Then add paper clips to the other pan, one at a time, counting as you go, until the pans balance.

Remind students that a protractor weighs 31 paper clips in the first pan balance. All that remains is to find the ruler’s weight in paper clips.

Ask: How can you change the pans to isolate the ruler in one pan, balanced by paper clips in the other pan? Tell students their solutions must follow one rule: Whatever you do, the pans must always remain balanced. Here are two possible approaches:

- Remove the protractor and replace it with 31 clips. Because the protractor weighs 31 clips, the pans remain balanced. Now remove 31 clips from each side. The pans remain balanced. So the ruler weighs as much as 39 paper clips.
- Remove the protractor from one pan. At the same time, remove 31 clips from the other pan. Because the protractor weighs 31 clips, the pans remain balanced. The ruler is balanced by 39 clips.

Example 2:
Show the ballpoint pens, compasses, and paper clips. Tell students their goal is to find the weight of a pen and the weight of a compass in terms of paper clips.

Set up the first pan balance. Place 2 pens in one pan. Then add clips to the other pan, one at a time, counting as you go, until the pans balance. (See margin.) Set up the second pan balance. Place 2 compasses in one pan. Place 1 compass and 1 pen in the other, and add paper clips, one at a time, counting as you go, until the pans balance. (See margin.)

Have students use the balances to test their solutions.

- Remove \( \frac{1}{2} \) of the objects from each pan of the first pan balance. The pans remain balanced. So 1 pen weighs 10 clips.

- Remove 1 pen from the second pan balance and replace it with 10 clips. The pans remain balanced.
### Solving Pan-Balance Problems

**Math Journal 2, pp. 336–338**

**Algebraic Thinking** Have partners complete the journal pages. Some of the problems show pictures of objects in the balance pans; some show squares and triangles; and some show expressions with letter variables.

Explain that some problems consist of two related parts and that students need to solve one of the parts before they have enough information to solve the other part. For example, in Problem 3, students need to complete the statement associated with the second pan balance before they can complete the statement associated with the first pan balance.

#### Problem 3

Complete the second statement first. One coin weighs as much as 7 marbles. Since 1 coin weighs as much as 7 marbles and 1 block and 1 coin weigh as much as 20 marbles, 1 block must weigh as much as 13 marbles. Remove the coin and 7 marbles from the pans of the first pan balance.

---

**Adjusting the Activity**

Have students write equations with variables to represent each of the pan-balance problems or match the problems to a prepared list of equations. They can record the correct equation above the illustration for each problem.

**Student Page**

<table>
<thead>
<tr>
<th>Problem</th>
<th>Illustration</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.</td>
<td><img src="image1.png" alt="Illustration" /></td>
<td>( y = 5x )</td>
</tr>
<tr>
<td>10.</td>
<td><img src="image2.png" alt="Illustration" /></td>
<td>( A = 3B )</td>
</tr>
<tr>
<td>11.</td>
<td><img src="image3.png" alt="Illustration" /></td>
<td>( 2y = 5z )</td>
</tr>
<tr>
<td>12.</td>
<td><img src="image4.png" alt="Illustration" /></td>
<td>( 10B = 12C )</td>
</tr>
<tr>
<td>13.</td>
<td><img src="image5.png" alt="Illustration" /></td>
<td>( 4A = 6B )</td>
</tr>
<tr>
<td>14.</td>
<td><img src="image6.png" alt="Illustration" /></td>
<td>( 1.5A = 2.5B )</td>
</tr>
<tr>
<td>15.</td>
<td><img src="image7.png" alt="Illustration" /></td>
<td>( 3.5A = 5B )</td>
</tr>
<tr>
<td>16.</td>
<td><img src="image8.png" alt="Illustration" /></td>
<td>( 5.5A = 8B )</td>
</tr>
</tbody>
</table>

---

**Adjusting the Activity**

Draw a new pan balance underneath the one on the board to illustrate each step. Refer to the Math Message Follow-Up, and replace each ball with 5 marbles.

Then draw another pan balance underneath to show each side of the balance divided in half. Cross off one half on each side.

Encourage students to begin by deciding which of the two statements should be completed first. For example, in Problem 3, the second statement needs to be completed first. However, in Problem 5, the first statement should be completed first.

Circulate and assist. A hint for Problem 12 on journal page 338 is to determine the weight of the coffee separate from the cup.
Making and Interpreting Line Plots

(Math Journal 2, p. 339)

Students make and interpret a line plot consisting of a data set of measurements obtained from fraction cards. Students plot fractional units on the number line from 0 through 1. Remind students that they can use benchmarks to place the fractions on the number line. Review fraction operations as needed to find the data landmarks.

Math Boxes 10-2

(Math Journal 2, p. 340)

Mixed Practice Math Boxes in this lesson are paired with Math Boxes in Lesson 10-4. The skill in Problem 5 preview Unit 11 content.

Writing/Reasoning Have students write a response to the following: Explain your solution strategy for Problem 4a.

Sample answer: The area of the rectangle is $3 \times 2.5 = 7.5$ ft$^2$.

The area of a triangle is $\frac{1}{2}$ the base times the height. Since the base for both figures is the same, and $\frac{1}{2}$ of the height of the triangle is greater than the height of the rectangle, the triangle’s area is greater because $3 \times 2.5 = 7.5$ ft$^2$.

Study Link 10-2

(Math Masters, p. 297)

Home Connection Students solve pan-balance problems with one and two balances.

3. Differentiation Options

READINESS

Solving Logic Puzzles

To provide experience with deductive reasoning required to solve linear equations, have students solve logic puzzles. Write the following statements on the board or the Class Data Pad.

1. All $K$ are $M$.
2. All $M$ are $B$.
In each figure below, the two pans are in perfect balance. Solve these pan-balancing problems.

1. One triangle weighs as much as 3 balls.

2. All weighs as much as 10 marbles.

3. One orange weighs as much as 2 marbles.

4. One cup of juice weighs as much as 6 blocks.

5. One apple weighs as much as 4 marbles.

Fill in the missing numbers to make true sentences.

6. \( \frac{7}{4} \times 2 = 2 \)

7. \( \frac{28}{7} \times \frac{9}{8} = 2 \)

8. \( (14 - 3) \times \frac{14}{2} = 6 \)

9. \( (3 - 3) \times (34 / 2) = 10 \)

10. \( 50 = \frac{7}{4} \times 14 - 6 \)

Practice.

Measuring Time

Franz buys two sandglasses from an antique dealer. However, when he gets home he realizes the sand in the sandglasses does not measure 1 hour. The first sandglass measures a nine-minute interval, and the other sandglass measures a thirteen-minute interval.

Franz wants to make a special cleaning solution to clean his new sandglasses. The solution needs to boil for 30 minutes. Can Franz use his sandglasses to measure 30 minutes from the time the solution starts to boil?

Explain your solution by describing what Franz should do.

When the solution starts to boil, set the 13-minute sandglass. When it runs out, 13 minutes will have passed. Set both the 13-minute and the 9-minute sandglasses. When the 9-minute sandglass runs out, 13 + 9 = 22 minutes will have passed. Start the 9-minute sandglass again. When the 13 minute sandglass runs out, 13 + 9 + 4 = 26 minutes will have passed. The 9-minute sandglass will have 4 minutes of sand in its bottom. Start the 9-minute sandglass again. Now the 4 minutes of sand are in the top. When the 9-minute sandglass runs out, 13 + 9 + 4 + 4 = 30 minutes will have passed.

To further explore solving a problem with two unknowns, have students use a sandglass model to solve the problem on the Math Masters page. Students determine whether a sandglass with a 9-minute interval and one with a 13-minute interval can be used to time a 30-minute period.

When partners have finished, discuss their solution strategies. One approach is to begin by setting the 13-minute sandglass. When it runs out, set the 13-minute and the 9-minute sandglasses at the same time. When the 9-minute sandglass runs out (13 + 9 minutes), start it again. After 4 more minutes, the 13-minute sandglass will run out (13 + 9 + 4), and there will be 4 minutes in the bottom of the 9-minute sandglass. Turn the 9-minute sandglass again so that the 4 minutes of sand on the bottom is now on the top. When this runs out, 30 minutes will have passed (13 + 9 + 4 + 4 = 30 minutes).