**Volume of Rectangular Prisms**

**Objective** To provide experiences with using a formula for the volume of rectangular prisms.

### Key Concepts and Skills
- Use formulas \((l \times w \times h)\) or \((B \times h)\) to calculate the volumes of rectangular prisms. [Measurement and Reference Frames Goal 2]
- Define the base and height of a rectangular prism. [Geometry Goal 2]
- Explore the properties of rectangular prisms. [Geometry Goal 2]
- Write number sentences with variables to model volume problems. [Patterns, Functions, and Algebra Goal 2]

### Key Activities
Students discuss the difference between volume and area. They define base and height for a rectangular prism and develop a formula for the volume of a rectangular prism. Students use a formula to find volumes and to model a rectangular prism with a given volume.

### Ongoing Assessment: Informing Instruction
See page 751.

### Key Vocabulary
- volume
- cubic unit
- rectangular prism
- face
- base (of a rectangular prism)
- height (of a rectangular prism)
- Associative Property of Multiplication

### Materials
- Math Journal 2, pp. 321, 322, and Activity Sheet 8
- Student Reference Book, pp. 195–197
- Study Link 9-7
- scissors
- transparent tape
- 36 centimeter cubes
- slate

### Advance Preparation
For Part 1, make models of the open boxes on Activity Sheet 8 to illustrate where to fold and where to tape the patterns for Boxes A and B. Have students save the boxes for use in Project 9.

Math Message

Write 2 questions that can be answered by reading Student Reference Book, page 195.

Math Message Follow-Up

(Student Reference Book, p. 195)

Ask students to pose their questions for the class. Use the questions and responses to discuss the difference between area and volume. To support English language learners, write the key ideas on the board. Emphasize the following points:

- Shapes that are 2-dimensional are flat. The surfaces they enclose take up a certain amount of area, but they do not have any thickness, so they do not take up any space.
- Shapes that are 3-dimensional have length, width, and thickness, so they enclose a certain amount of space.
- The amount of surface inside a 2-dimensional shape is the area of the shape. Area is measured in square units, such as square inches, square feet, square centimeters, and so on.
- The amount of space enclosed by a 3-dimensional shape is the volume of the shape. Volume is measured in cubic units, such as cubic inches, cubic feet, cubic centimeters, and so on.
- The symbol used to indicate square units is the abbreviation of the unit name with a superscript 2, for example, in², ft², cm², m², and so on. For cubic units, the symbol is the abbreviation of the unit name with a superscript 3, for example, in³, ft³, cm³, m³, and so on. To support English language learners, discuss the meanings of the word volume. Students might have heard the word used in contexts involving sound level or a book. Emphasize that volume is also used in the mathematical context involving 3-dimensional shapes.
Defining Base and Height for Rectangular Prisms

(Math Journal 2, p. 321)

As a class, read the introduction on journal page 321. Have students study the figures and then write their own definitions for the base and height of a rectangular prism. Ask volunteers to share their definitions with the class. Use their responses to reinforce the following points:

- A rectangular prism is formed by six flat surfaces, or faces, that are rectangles. To support English language learners, compare the common meaning and the mathematical meaning of the word face.
- Any of the six faces of the prism can be a base, but the face that the prism “sits on” is often chosen as a base.
- The height (for a given base) is the shortest distance between the base and the opposite face.

Links to the Future

The names and properties of prisms will be revisited in more detail in Unit 11.

There are two types of prisms, right and oblique. (See below.) When the faces of a prism are all rectangles, it is a right rectangular prism. When the faces are not all rectangles, it is an oblique rectangular prism. Oblique prisms are not considered in Fifth Grade Everyday Mathematics.

Adjusting the Activity

Have students identify objects in the classroom that have the shape of rectangular prisms.

Developing a Formula for Volume

(Math Journal 2, pp. 321, 322, and Activity Sheet 8; Student Reference Book, pp. 196 and 197)

Algebraic Thinking Give 24 centimeter cubes to each student. Ask students to tear out Activity Sheet 8 from the back of their journals and follow the directions on journal page 321 to answer Problems 2 and 3.
When students have completed the problems, display your prepared models as guides for folding and taping the boxes. Direct students to do the following:

1. Cut out the pattern for Box A, fold it on the dashed lines, and tape it to make an open box.

2. Cover the bottom of Box A with one layer of centimeter cubes. Ask: How many cubes are in this layer? 8

3. Put a second layer of cubes on top of the first layer. Ask: How many cubes are in the box now? 16

4. Put a third layer on top of the second layer. Ask: How many cubes are in the box now? 24 What is the volume of the box? 24 cm³

5. Cut out the pattern for Box B, fold it on the dashed lines, and tape it to make an open box.

6. Cover the bottom of Box B with one layer of cubes. Ask: How many cubes are in this layer? 9

Ask students to answer the following questions without putting any more cubes in the box:

- How many layers of cubes are needed to fill the box? 5
- How many centimeter cubes in all are needed to fill the box? 45
- How did you find the answer? Sample answer: By multiplying the number of cubes in 1 layer by the number of layers.
- What is the volume of the box? 45 cm³

Point out that the number of cubes in one layer is the same as the number of square centimeters in the base (l * w), and that the number of layers is the same as the height in centimeters of the box. Ask students how this information might be used in a formula to find the volume of the box. Multiply the area of the base by the height of the box.

This formula is written as $V = B \times h$, where $V$ represents the volume, $B$ represents the area of the base, and $h$ represents the height from that base. Ask students to write this formula at the top of journal page 322. Have students refer to pages 196 and 197 of the Student Reference Book as needed.
Another Formula for Volume of a Rectangular Prism
(Student Reference Book, pp. 196 and 197)

Ask students to review pages 196 and 197 of the Student Reference Book. Give each student 12 more centimeter cubes so each student has 36 cubes.

**NOTE** If you do not have 36 centimeter cubes per student, ask students to build the rectangular prisms in partnerships.

Ask students to use the centimeter cubes to do the following:

1. Build a rectangular prism with a base that is 4 cubes long and 3 cubes wide, and that is 1 cube high.
   Ask: Describe a way you could determine the volume of the rectangular prism. Sample answer: Multiply 4 * 3 * 1 to obtain 12 cm³. What do the 4, 3, and 1 represent in the prism? The length, width, and height.

2. Build a second 4-by-3-by-1 rectangular prism. Place it on top of the original rectangular prism.
   Ask: How could you determine the volume of the new rectangular prism? Sample answers: Multiply 4 * 3 * 2 to obtain 24 cm³; multiply 12 * 2 to get 24 cm³.

3. Build a third 4-by-3-by-1 layer. Place it on top of the existing rectangular prism. Ask students to write a number model using three factors to find the volume of the new prism.
   4 * 3 * 3 = 36 cm³

Now ask the following question: Suppose you know the number of cubes in the length, width, and height of a rectangular prism. How could this information be used to write another formula for finding the volume of a rectangular prism? Multiply: length * width * height (l * w * h); V = l * w * h.

Have students write this formula at the top of journal page 322, next to the first formula they derived for finding the volume of a rectangular prism. Discuss why both formulas will result in the same volume.

Conclude by asking students to explain how they could use the new formula to find the volume of a cube. Sample answers: Replace each side-length letter in the formula with the length of a side of the cube. You could rewrite the formula as \( V = s^3 \), where \( s \) is the length of a side of the cube.
**Building a Rectangular Prism with a Given Volume**

**Algebraic Thinking** Ask students to use centimeter cubes to build the shape of a rectangular prism with a volume of 24 cm³.

Ask: *What are the possible areas for the base (B) and the length of the height (h) from that base?* 

- $B = 24 \text{ cm}^2, h = 1 \text{ cm}$; $B = 12 \text{ cm}^2, h = 2 \text{ cm}$; $B = 8 \text{ cm}^2, h = 3 \text{ cm}$; $B = 6 \text{ cm}^2, h = 4 \text{ cm}$; $B = 4 \text{ cm}^2, h = 6 \text{ cm}$; $B = 3 \text{ cm}^2, h = 8 \text{ cm}$; $B = 2 \text{ cm}^2, h = 12 \text{ cm}$; $B = 1 \text{ cm}^2, h = 24 \text{ cm}$.

Ask students to use the cubes to make a rectangular prism that has a length of 3 cm, a width of 2 cm (base of 6 cm²), and a height of 4 cm. Explain that the volume can be found as follows:

$$V = (l \times w) \times h = (3 \times 2) \times 4$$

Now ask students to carefully rotate their rectangular prism so that it now has a length of 4 cm, a width of 2 cm (base of 8 cm²), and a height of 3 cm. Explain that the volume can be found as follows:

$$V = (l \times w) \times h = 3 \times (2 \times 4)$$

Ask: *What do the two rectangular prisms have in common?* Sample answer: The dimensions of the prisms have the same three measures, and each has a volume of 24 cm³.

Guide students to conclude that if the measures of the three dimensions of two rectangular prisms are the same, the volumes are the same. Then point out that this is an illustration of the **Associative Property of Multiplication**, which means that changing the grouping of factors does not change the product.

$$\text{So, } (3 \times 2) \times 4 = (3 \times (2 \times 4))$$

$$6 \times 4 = 3 \times 8$$

$$24 = 24$$

Have students repeat this activity by comparing the volumes of two rectangular prisms that have dimensions of 2 cm, 3 cm, and 5 cm, but different bases.

**Finding the Volumes of Rectangular Prisms**

(From Math Journal 2, p. 322)

**Algebraic Thinking** Have students complete journal page 322 by calculating the volume of six rectangular prisms from the given dimensions.
LESSON 9-8

**Math Boxes**

1. Solve.
   - a. \( \frac{3}{5} \times \frac{1}{8} = \frac{3}{40} \)
   - b. \( \frac{3,840}{4} = 960 \)
   - c. \( \frac{3,840}{4} \times \frac{1}{2} = \frac{960}{2} = 480 \)
   - d. \( \frac{3}{8} \times \frac{5}{6} = \frac{15}{48} = \frac{5}{16} \)

2. Complete the “What’s My Rule?” table, and state the rule.

<table>
<thead>
<tr>
<th>Rule</th>
<th>in</th>
<th>out</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \div 30 )</td>
<td>254.04</td>
<td>8.47</td>
</tr>
<tr>
<td>( \times 30 )</td>
<td>360.00</td>
<td>6.00</td>
</tr>
<tr>
<td>( \div 30 )</td>
<td>1,350.00</td>
<td>45.00</td>
</tr>
<tr>
<td>( \times 30 )</td>
<td>2,190.00</td>
<td>73.00</td>
</tr>
</tbody>
</table>

3. Use the graph to answer the questions.
   - a. Which day had the greatest attendance? Friday
   - b. What was the total attendance for the five-day period? 90

4. Elena received the following scores on math tests: 80, 85, 76, 70, 87, 80, 90, 80, and 90.
   - Find the following landmarks:
     - maximum: 90
     - minimum: 70
     - range: 20
     - mode: 80
     - mean: 82

5. Ongoing Learning & Practice

   - **Math Boxes 9-8**
     
     (Math Journal 2, p. 323)

   - **Mixed Practice** Math Boxes in this lesson are paired with Math Boxes in Lesson 9-10. The skills in Problems 2 and 5 preview Unit 10 content.

   - **Writing/Reasoning** Have students write a response to the following: Explain the strategy you used to solve Problem 1c and explain your reasoning. Answers vary.

6. **Ongoing Assessment: Recognizing Student Achievement**

   Use Math Boxes, Problem 3 to assess students’ abilities to find common denominators. Students are making adequate progress if they correctly identify the least common denominators.

   [Number and Numeration Goal 5]

7. **Study Link 9-8**

   (Math Masters, p. 279)

   - **Home Connection** Students find the volume of cube structures.
**Differentiation Options**

### Analyzing Prism Nets for Cubes

*Math Masters, pp. 280 and 429*

To stimulate students’ ability to visualize, name, and describe geometric solids, have students look at diagrams to determine which shapes can and cannot be folded into cubes. Read and discuss the introduction as a group. As students choose which of the diagrams can be folded into a cube, provide inch grid paper so students may check their work.

Discuss students’ solutions.

### Finding the Volume of One Stick-On Note

*Math Masters, p. 281*

To apply students’ understanding of how to find the volume of a rectangular prism, have partners compare the volume of a single stick-on note and that of a centimeter cube. Give each partnership one stick-on note, one unused pad of stick-on notes, and one centimeter cube. Ask students to estimate how the volume of the single stick-on note compares with the volume of the cube.

Have students record their strategies and solutions on *Math Masters*, page 281. When partners have completed the page, ask them to present their solutions.

**NOTE**
One approach to finding the volume of a single stick-on note would be to measure the dimensions and find the volume of the unused pad of stick-on notes. A single stick-on note would represent a fraction of the pad.

### 5-Minute Math

To offer students more experience with calculating the volumes of prisms, see *5-Minute Math*, pages 52 and 214.