Objective  To develop a fraction multiplication algorithm.

Key Concepts and Skills
- Use an area model to find fractions of fractions. [Operations and Computation Goal 5]
- Write number models for fraction multiplication problems shown with an area model. [Operations and Computation Goal 7]
- Describe the patterns in the area model for fraction multiplication. [Patterns, Functions, and Algebra Goal 1]
- Recognize the patterns in products when a number is multiplied by a fraction that is less than or equal to 1. [Patterns, Functions, and Algebra Goal 1]

Key Activities
Students solve problems using an area model for fraction multiplication. They use the area model and fraction multiplication patterns to derive the standard algorithm.

Ongoing Assessment: See page 651.
- Informing Instruction
- Recognizing Student Achievement

Math Boxes 8-6
Math Journal 2, p. 267
Geometry Template
Students practice and maintain skills through Math Box problems.

Study Link 8-6
Math Masters, p. 232
Students practice and maintain skills through Study Link activities.

Key Vocabulary
area model

Materials
Math Journal 2, pp. 264–266
Student Reference Book, p. 76
Study Link 8-5; transparency of Math Masters, p. 231 (optional) • slate

Advance Preparation
For Part 1, make a transparency of Math Masters, page 231 for use in the discussion of journal page 265. The master is identical to the journal page.

Teacher’s Reference Manual, Grades 4–6  pp. 143, 144, 220, 221
1. Use the rectangle at the right to sketch how you would fold paper to help you find $\frac{1}{3}$ of $\frac{2}{3}$.

What is $\frac{1}{3}$ of $\frac{2}{3}$?

2. Use the rectangle at the right to sketch how you would fold paper to help you find $\frac{1}{4}$ of $\frac{3}{5}$.

What is $\frac{1}{4}$ of $\frac{3}{5}$?

3. Rewrite $\frac{2}{3}$ of $\frac{3}{4}$ using the multiplication symbol $\ast$.

4. Rewrite the following fraction-of-fraction problems using the multiplication symbol $\ast$.

- $\frac{1}{2} \ast \frac{1}{3}$
- $\frac{2}{3} \ast \frac{3}{4}
- \frac{5}{6} \ast \frac{2}{3}$
- $\frac{3}{5} \ast \frac{5}{2}$

Study Link 8-5 Follow-Up

Discuss students’ solution strategies for Problem 5.

Teaching the Lesson

Math Message Follow-Up

(Math Journal 2, p. 264)

Ask a volunteer to demonstrate how to fold and shade a sheet of paper to solve the Math Message problem. Reinforce the demonstration by sketching the folds on the board or a transparency. The folded paper and sketch should show the paper folded into thirds in both directions. The whole has 9 parts, 2 of them shaded twice, so the answer is $\frac{2}{9}$. Have students complete Problem 2.

Introducing the Word of to Mean Multiply

(Math Journal 2, p. 264)

Ask students what operation they used to solve the Mental Math and Reflexes problems. multiplication Point out that the number of coins in 3 sets of 4 coins is equal to 3 times 4. Similarly, $\frac{2}{3}$ of $\frac{3}{4}$ is equal to $\frac{2}{3}$ times $\frac{3}{4}$. Students should record this as the answer to Problem 3 on the journal page. Have students complete the rest of the page and then compare answers with a partner.

Using the Area Model for Fraction Multiplication

(Math Journal 2, p. 265)

On the board or the transparency of Math Masters, page 231, demonstrate how to solve $\frac{5}{6} \ast \frac{2}{3}$ from Problem 1 on journal page 265. Partition and shade a rectangle. Students should reproduce the shading in the rectangle at the top of the journal page.
1. Note that the denominator of the second fraction is 4.
2. Divide the rectangle into fourths vertically. (See Figure 1.)
3. Shade the left \( \frac{3}{4} \) of the interior of the rectangle.
4. Note that the denominator of the first fraction is 3.
5. Divide the rectangle into thirds horizontally. (See Figure 2.)
6. Use a different shading to shade the bottom \( \frac{2}{3} \) of the interior.
7. Note the parts that have been shaded twice.

**Ongoing Assessment: Informing Instruction**

Watch for students who have difficulty sketching the area models for the problems on the journal page. Have them first use the folded-paper method and then copy the results to the journal page.

Discuss the results:

- How does the diagram show the answer to \( \frac{2}{3} \times \frac{3}{4} \)? The parts of the rectangle that were shaded first represent \( \frac{3}{4} \) of the rectangle. The parts that were shaded second represent \( \frac{2}{3} \) of the rectangle. The parts that have both shadings represent \( \frac{2}{3} \times \frac{3}{4} \) of \( \frac{3}{4} \) of the rectangle. 6 parts out of 12 are shaded twice, so \( \frac{2}{3} \times \frac{3}{4} \) or \( \frac{2}{3} \times \frac{3}{4} \) equals \( \frac{6}{12} \) or \( \frac{1}{2} \).
- How is this model similar to paper folding? The lines that divide the rectangle into fractional parts are just like the folds that divide the paper into fractional parts.
- Why would either of these models be called an **area model** for fraction multiplication? The doubly shaded part represents a fraction. The area of the doubly shaded part of the rectangle is a fraction of the area of the whole rectangle.

Have partners complete the journal page. Circulate and assist. When most students have finished, briefly review the answers.

**Ongoing Assessment: Recognizing Student Achievement**

Use **Journal Page 265** to assess students’ understanding of the area model for fraction multiplication. Students are making adequate progress if their sketches accurately reflect the correct answers.

[Operations and Computation Goal 5]
After briefly reviewing answers, pose a few fraction multiplication number stories. Suggestions:

- Laura and her brother will paint \( \frac{3}{4} \) of a wall. How much of the entire wall will each paint if they share the work equally? \( \frac{3}{8} \) of the wall
- A tube of blue oil paint holds \( \frac{5}{8} \) of an ounce of paint. It takes \( \frac{3}{4} \) of the tube to paint the background of a sign. How many ounces of blue paint does it take to paint the background? \( \frac{15}{24} \), or \( \frac{5}{8} \) oz

Deriving a Fraction Multiplication Algorithm

(Math Journal 2, p. 266; Student Reference Book, p. 76)

Have students work on Problems 1 and 2 on journal page 266. Discuss their answers. Students should notice that the product of the numerators is equal to the numerator of the product and that the product of the denominators is equal to the denominator of the product.

Ask volunteers to share their descriptions of a fraction multiplication algorithm from Problem 2. Explain that the relationships and descriptions are examples of the following general pattern: To multiply fractions, simply multiply the numerators and multiply the denominators. This pattern can be expressed as \( \frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} \). Remind students that \( b \) and \( d \) cannot be 0. Ask students to complete the journal page using this algorithm for fraction multiplication. Ask students to refer to Student Reference Book, page 76 as needed.

Circulate and assist. When most students have finished, have partners share answers and resolve differences.

NOTE Fraction multiplication is deceptively easy. One can easily learn by rote that to multiply fractions, one simply multiplies the numerators and multiplies the denominators. However, through modeling fractions by folding paper and shading rectangles students internalize concepts necessary to understand the meaning of expressions like \( \frac{3}{4} \times \frac{2}{3} \).

When students do not understand the reasons behind the fraction multiplication algorithm, they are more likely to add (or subtract or divide) numerators and denominators when they add (or subtract or divide) fractions. Students who understand what to do and why they are doing it will be better able to apply their knowledge in problem solving.
Ask students to revisit Problem 1 on journal page 265. Remind them that they began by dividing the rectangle into fourths vertically and shading three of those four parts to show $\frac{3}{4}$ of the rectangle shaded. Then they divided the rectangle into thirds horizontally. Ask:

- How does dividing the rectangle into thirds (before applying the second round of shading) affect the number of parts in the rectangle and the number of parts that were shaded initially?  
  *Sample answer: Dividing the rectangle into thirds resulted in 3 times as many parts in all and 3 times as many parts shaded.*

- Did this change how much of the rectangle was shaded?  
  *Explain. Sample answer: No, because the resulting fraction, $\frac{9}{12}$, is equivalent to the original fraction, $\frac{3}{4}$; the shaded amount of the rectangle remained the same.*

Repeat the above discussion for Problem 5 ($\frac{3}{8} \times \frac{3}{5}$) on journal page 265. Students should conclude that after the rectangle was divided into eighths horizontally, there were now 8 times as many parts in all and 8 times as many parts shaded. So, the $\frac{3}{5}$ of the rectangle that was initially shaded became $\frac{24}{40}$. Because $\frac{3}{5} = \frac{24}{40}$, the fractional part of the rectangle that was initially shaded did not change.

Help students conclude that when they divide rectangles into equal parts horizontally (or vertically if they began with horizontal partitions), they are in essence multiplying the numerator and the denominator of the original fraction by the same (nonzero) number.

Explain that this pattern can be expressed as $\frac{a}{b} = \frac{n \times a}{n \times b}$, where $b \neq 0$ and $n \neq 0$. Multiplying by $\frac{n}{n}$ does not change the value of the original fraction because it is equivalent to multiplying the fraction by 1. Ask students to try out this pattern on Problem 6.

Pose the following problems to provide additional practice multiplying by a fraction that has a value of 1.

- $1 \times \frac{3}{4} = \frac{3}{4}$
- $\frac{4}{9} \times \frac{5}{9} = \frac{20}{81}$
- $\frac{5}{8} \times \frac{3}{4} = \frac{15}{32}$
- $\frac{4}{7} \times \frac{7}{4} = \frac{1}{10}$
### Multiplying by a Fraction Less than 1

*(Math Journal 2, pp. 265 and 266)*

Have students examine the drawings and the results in Problems 1–7 on journal page 265. Ask the following questions:

- In each problem, does the fractional part of the rectangle you shaded first represent the first fraction or the second fraction of the problem? It represents the second fraction.
- In each problem, how does the first fraction compare to 1? The first fraction in each problem is less than 1.
- In each problem, how would you compare the fractional part of the rectangle with double shading to the fractional part you shaded first? Sample answer: In each case, the fractional part with double shading is a portion of what I shaded first. So it is less than what I shaded first.
- In each problem, how does the product compare to the second fraction? Explain why that is the case. Sample answer: Each product is less than the second fraction. In each problem, we multiplied the second fraction by a fraction that is less than 1. So we were finding a portion of the second fraction.

Ask students to examine the results in Problem 3 on journal page 266. Ask: *Do the results support what we said about a fraction being multiplied by a number less than 1?* Yes Ask students to describe the size of the product when you multiply a number by a fraction that is less than 1. Sample answer: When you multiply a number by a fraction that is less than 1, the product is less than the original number. For example, in Problem 3a, the product is a fraction, \( \frac{1}{15} \), that is less than the original fraction, \( \frac{1}{5} \).

### Ongoing Learning & Practice

#### Math Boxes 8-6

*(Math Journal 2, p. 267)*

**Mixed Practice** Math Boxes in this lesson are paired with Math Boxes in Lesson 8-8. The skill in Problem 5 previews Unit 9 content.

#### Study Link 8-6

*(Math Masters, p. 232)*

**Home Connection** Students solve fraction multiplication problems. They use the area model and the fraction multiplication algorithm.
Supporting Fraction Multiplication Concepts

(Math Masters, p. 233)

To provide an additional model for the connection between the word of and multiplication, guide students through the activity on the Math Masters page.

Problem 1 Students review using multiplication to find the area of a rectangle and an area that is a fractional part of the rectangle. Point out that the numerators represent the length and width of the part of the rectangle that is shaded. Remind students that $\frac{1}{3}$ is equivalent to $\frac{2}{4}$. The denominators represent the length and width of the entire rectangle. The product of the fractions names the twice-shaded area of the rectangle.

Problem 2 Students find and shade a similar fractional part of a circle.

Review both problems by emphasizing that finding $\frac{1}{3}$ of $\frac{1}{2}$ of the rectangle is the same as finding $\frac{1}{3}$ of $\frac{1}{2}$ of the circle. Both problems can be solved using multiplication.

Pose several problems for students to solve using multiplication, and have them write the number sentences.

- $\frac{1}{5}$ of $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{20}$
- $\frac{2}{5}$ of $\frac{3}{4}$ of $\frac{1}{2} = \frac{3}{20}$
- $\frac{2}{5}$ of $\frac{3}{4}$ of $\frac{2}{3} = \frac{6}{20}$ or $\frac{3}{10}$

Fraction Problems

(Math Masters, p. 234)

Students solve real-world number stories involving fractions. They are asked to first write an open number model for each problem.
Fraction Problems

1. Ailene is baking corn bread. She will cover \(\frac{3}{4}\) of the cornbread with cheese. Then she plans to give \(\frac{2}{3}\) of the cornbread with cheese to her friend Alex.
   a. Use the rectangle to show an area model for the problem.
   b. Write an open number model for the problem. Choose a letter to stand for the portion that will be given to Alex. 
   c. Ailene will give ______ of the cornbread to Alex.

2. A recipe for granola bars calls for \(\frac{1}{2}\) cup almonds. Cy is making \(\frac{3}{4}\) of the recipe.
   a. Write an open number model to show how many ounces of almonds Cy will use.
   b. Cy will use ______ cup of almonds.

3. An ant weighs \(\frac{1}{10}\) the weight of a crumb that it is carrying. Suppose the crumb weighs \(\frac{3}{100}\) gram.
   a. Write an open number model to show the weight of the ant in grams.
   b. The ant weighs ______ gram.

4. Walker plans to hike a trail that is \(\frac{8}{10}\) of a mile long. So far, he has walked \(\frac{1}{4}\) that distance.
   a. Write an open number model for the problem.
   b. So far, Walker has walked ______ mi.

5. In Mrs. Ortiz’s class, \(\frac{9}{22}\) of the students are boys. Of the boys, \(\frac{1}{9}\) are left-handed.
   a. Write an open number model to show how to find what fraction of the class are left-handed boys.
   b. ______ of the class are left-handed boys.